Addressing the Lack of Persistence Among Students When They Are Faced With Challenging Math Problems

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Abstract

Lack of persistence when faced with challenging math problems impacts student achievement. Having taught high school math for only three years, I have observed its effect on students in my class every year. Some studies show the importance of entry points in form of questions positively contribute to a student's spending more time, able to positively struggle during the process of problem solving which extends their thinking that, in turn, counteracts lack of persistence. The purpose of this action research was to examine different strategies used in classrooms to decrease lack of persistence leading to an entry point into a challenging math problem and spending time in problem solving. During the extended thinking framework cycle, students participated in three strategies in their lessons and activities involving asking protocol-based questions and students responding while recording each others' responses during which they received written feedback. Data collection included pre- and post-intervention surveys student work, and observation data. Findings from the data suggested that the use of strategies that involve protocol questions when faced with challenging math problems increased students spending more time by persisting in challenging math problems.

Addressing the Lack of Persistence Among Students When They Are Faced With Challenging Math Problems

I teach 9th & 10th grade Algebra A & B at an Oakland charter high school. A large percentage of charter schools are single-site operations. This statistic is particularly profound in California where 656 schools, (55% of all charter schools) are single-site entities without central office services or staff. Unlike their traditional public school counterparts, charter schools go through a reauthorization process every five years (Cone & Kenda, 2015).

As a Math teacher for the past 2 ½ years, there has been an underlying obstacle among some of my math students. **Some students lack persistence when faced with challenging math problems.** Lack of persistence impacts some of my students' mathematics achievement on academic class and standardized assessments. In addition, when working on word problems, my student give up quickly instead trying to understand the problem. Many of my math students lack persistence in making sense of challenging math problems and do not persevere in solving them. They lack the persistence to **extend their thinking** by spending time trying ideas, making mistakes, applying strategies and reasoning deductively. Some observational reality data examples that I have observed in my class from students include:

- quits, give up
- leaves the math problem(s) incomplete.
- does not ask questions.
- does not talk about their answers
- does not check their answer(s)
- gets stuck

- does not draw a table, graph or what the word problem describes
- does not spend time on homework online
- feels like it takes too long to solve

Further to this point, it is clear by watching students choose other activities that are also challenging and requires persistence reveal that they do indeed know how to persist. Again here is my own observational reality data that support when students will persist in other extra curricular or curricular activities:

- boxing after school
- playing high school basketball boys team
- playing high school soccer girls/boys team
- playing high school volleyball girls team
- students stay after school for math tutoring voluntarily

Our Math Department had a general discussion along the lines of a major cause of poor student learning and performance, which is their tendency to quit -- to tune out during discussions of complex material and/or to give up on difficult assignments or tests. For instance, in my math class, at the micro level, some of my students will not even start a simple table with the names of people involved in a word problem. We discussed that it is difficult to discuss engagement with students at the macro level of not writing a paper or not solving a word problem. **Students claim to not understand a word problem, but they will reluctantly agree that they can start the table, and once the table is started, they admit that they can reread**

the problem to find numbers to put into the table.

Our math department then discussed the fact that most educators frame issues around content: **which content standards to teach**, where are the content gaps, how to scaffold the content, how to deliver the content with blended learning or better books.

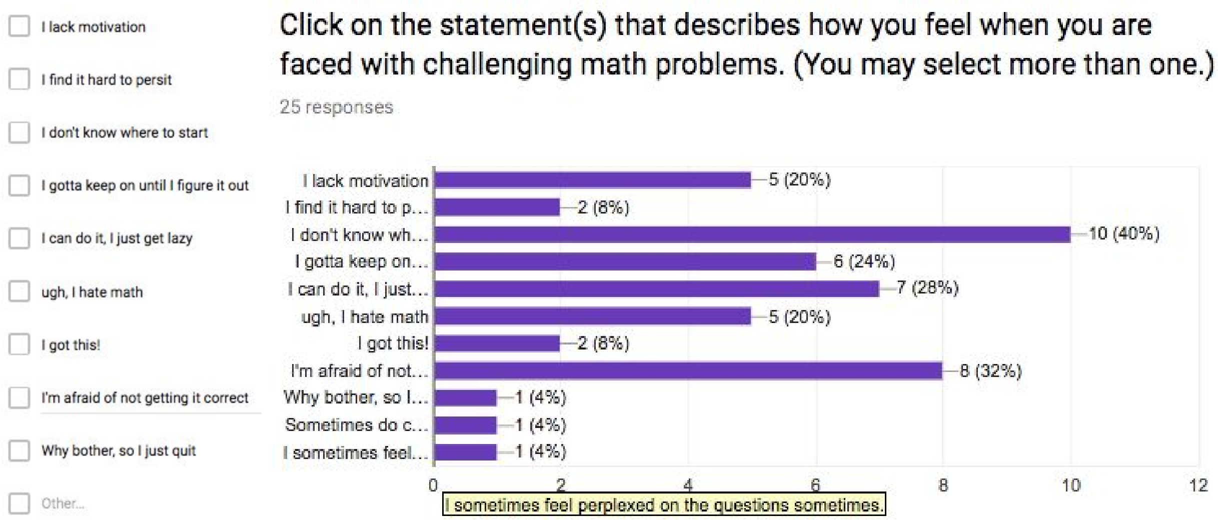
According to the NGA Center and CCSSO (2018) common core math practice standards, Make sense of problems and persevere in solving them, "Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution.They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need.

- Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem.
- Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches."

I am left wondering that maybe it is the lack of math proficiency within students who do not persist. According to an article that spoke about De-identifying with Math, Lambert saw a profound effect on students' sense of what constituted "being good" at math. Those who had taken pride in contributions to discussions and their persistence lost standing among their peers in terms of how their math skills were perceived. Even students good at memorization, she says, "talked about the anxiety they were feeling about mathematics." Several who had reported enjoying math found the subject "was no longer interesting to them." Lambert concluded that several students were "de-identifying with math" (Tulis, M., Fulmer, S., 2013).

In this same article, Allen says teachers gain by learning the source of students' feelings about math, including how their identities are shaped by community and family. "There is no other subject where parents come to you and say, 'Don't be too hard on our kid; we suck at math in our family,'" she observes. It's also problematic for black and Latino students that math is cast as a white and European activity. "Our black students don't have any sense that there are black folks who have contributed or aren't even aware that powerful mathematics have come out of Egypt" (Tulis, M., Fulmer, S., 2013).

As a baseline in discovering a lack of persistence perception and/or identity in my classes, I sent the following survey to 25 of my students in Algebra B. Here is reality data:



As one can see, only 8% of 25 students chose the statement, "I find it hard to persist". So indeed some students lack persistence when faced with challenging math problems. Out of 25 students, 20% chose that they "lack motivation". I was pleasantly surprised that 40% of students chose 'I don't know where to start'. In tandem to this statement, 32% chose, "I'm afraid of not getting it correct." Although 1% of students chose, "Why bother, so I just quit," 24% of student chose, " I gotta keep on until I figure it out".

A most outstanding statement-choice from students on the pre-survey from 40% of students was that of "I don't know where to start". If mathematically proficient students could start by explaining to themselves the meaning of a problem and looking for **entry points to its solution** this would impact their math learning whenever they are faced with challenging math problems.

In the article previously mentioned, according to a 6th & 7th grade study by Tulis & Fulmer (2013), the main purpose was to analyze the impact of changes in motivational and emotional states on students' **persistence**. Therefore, situational interest, task-related affect and specific emotion states (enjoyment, anger, anxiety and boredom) were measured at multiple time points before, during and after the task. The results of both studies emphasize the importance of situational interest for persistent engagement through challenge. Additionally, as a negative-activating emotion, slightly increasing anxiety throughout the task was found to be beneficial for **persistence**. In contrast, boredom (a negative deactivating emotion) turned out to be detrimental for **persistence** (Tulis, M., Fulmer, S. 2013).

My Math Department's Structure with Anecdotal Data:

I would like to preface that despite our math department using a controversial system of tracking, the focus of my action research is improving the persistence of my students towards problem solving thinking & application.

According my school's Math Department Lead, "Our students have been tracked by pre and post assessments during their summer school as soon as they start as incoming freshman from various other Oakland middle schools into our Summer Success Academy before the Academic Freshman Year. Created by our schools Math Department lead, Mr. M, a pre-diagnostic assessment called Unity's Basic 25, see Appendix B, and also translated in a Spanish version is given to those students to assess their prior knowledge on the following groups :

- Addition/Subtraction +/-
- Fractions
- Decimals
- Percents %
- Equations with variables

- Lines/Graphs
- Area
- Exponents
- Radians

The data from their initial assessment is recorded on a spreadsheet and later compared to a second version of the Unity's Basic 25 assessment taken at the end of the 4-week Summer Success Academy.

It is from this second assessment that a sort of assorting or 'tracking' of students from high skilled and low skilled is analyzed and then dispersed into two Math sections, Algebra 1 and Algebra A, offered during the academic freshman year. Here's the 4-year high school math courses trajectory for both types students who either score 70% higher or below:

Freshman year starting with Algebra A (scored 69% or below on the second version of the Unity's Basic 25 at the end of the 4-week Summer Success Academy): ❑ Freshman year - Algebra A ❑ Sophomore year - Algebra B ❑ Junior year - Algebra 2 & Geometry ❑ Senior year - Pre-calculus or Calculus (optional but strongly insisted)	Freshman year starting with Algebra 1(scored 70% or higher on the second version of the Unity's Basic 25 at the end of the 4-week Summer Success Academy): ❑ Freshman year - Algebra 1 ❑ Sophomore year - Algebra 2 or Algebra 2 Honors & Geometry ❑ Junior year - Pre- calculus (optional but strongly insisted) ❑ Senior year - Calculus (optional but strongly insisted)

According to our Math department lead, "A major reason for splitting Algebra 1 into two courses was to give our weaker students more time to absorb the material. Even though our students are placed in either Algebra A or Algebra 1 to start off their Freshman academic year, both sections will incur students moving up or down depending on their effort to persist/push through the material and/or exceed what is given. Within the first 4 weeks of the academic year, freshman in Algebra A have an opportunity to move up into the Algebra 1 section if they fulfill the following requirements:

- Grades are 90% or higher
- Student shows a desire to learn more than what is required
- Student exhibits acquisition of concepts easily and can explain/tutor other students

When given this opportunity to move up, still **students lack persistence when faced with challenging math problems.** The math department lead attributed student lack of persistence to several factors, including:

- School work is something I do just to show what I know, not something I do to expand what I know.
- If I do not know how to start, or if I do not remember the steps I learned, I cannot do the problem, so it is a waste of time to even start.
- I don't know how to start because I am stupid, so avoiding the problem will diminish my shame.

Yet, dwelling significantly is my problem of practice, in that, students lack persistence when faced with challenging math problems.

Literature Review

Introduction

In this literature review, I will be discussing the definition of persistence and challenging math tasks, differentiating between persistence and motivation, what literature says in regards to three overarching questions and ways where lack of persistence have been addressed leading into my chosen intervention.

Definition of Persistence and Challenging Math Tasks

We use the term persistence to describe the category of student actions that include concentrating, applying themselves, believing that they can succeed, and **making an effort to learn**; and we term the **tasks** that are likely to foster such actions challenging, in that they allow the possibility of **sustained thinking**, decision making, and some risk taking by the students.(Clarke, Roche, Cheeseman, Van der Schans, Mathematics Teacher Education & Development, 2014/2015).

Furthermore, Sullivan, Cheeseman, Michels, Mornane, Clarke, Roche, and Middleton (2011) characterised challenging tasks as those that require students to:

• plan their approach, especially sequencing more than one step;

• process multiple pieces of information, with an expectation that they make connections between those pieces, and see concepts in new ways;

• engage with important mathematical ideas;

• choose their own strategies, goals, and level of accessing the task;

• spend time on the task;

• explain their strategies and justify their thinking to the teacher and other students; and

• extend their knowledge and thinking in new ways (p. 34) (Clarke, Roche, Cheeseman, Van der Schans, 2014/2015).

In recent years, the mathematics education research community has given increased focus to the use of cognitively demanding, **challenging tasks** and the demands placed on students and teachers by their use. In particular, there is evidence that a major issue is students' **lack of persistence** when working on such tasks (Clarke, Roche, Cheeseman, Van der Schans, 2014/2015).

Differentiating Persistence vs. Motivation

A colleague within this program challenged me on how would I know it was not lack of motivation as opposed to lack of persistence. According to Appendix A, The Motivation and Engagement Wheel, Figure 3 by Sinicrope, R., Eppler, M., Preston, R., Ironsmith, M., (2015), whether adaptive motivation or maladaptive motivation, both are one quarter sections of the wheel that involve such slices as self-efficacy, mastery orientation, valuing or maladaptively, such slices as uncertain control, failure avoidance and anxiety. None of those slices ever collide in the other quarter sections. Those sections called Adaptive Engagement and Maladaptive Engagement are sections that do not share connections to that of motivation. As one can see in Appendix A, persistence is part of Adaptive Engagement and not part of any section that involves motivation. I can safely move forward that motivation is quite different to persistence and it's roots lack thereof derive from different spectrums.

Overarching Questions

One question that I plan to address is, 'What is the nature of the relationship between students' persistence and challenging/non-challenging math problems?' Research suggests that the typical student has little interest in math after the 4th grade (Pogrow, Sept. 94). Students who are turned off by math are also likely to be rebelling against the notion of adulthood -- or at least the notion of being like most of the adults they know. Such students think that they are different, immortal, and can easily achieve anything they want. The worst thing that you can do to students who think that math is a set of pointless, adult-imposed rules is to tell them that they will understand the need for math when they grow up or that learning math will make them more successful adults. (Pogrow, Dec. 2004)

Central to our math disconnect, say education experts, are misconceptions reinforced in classroom practice, including emphasis on finding "the answer"—fast. "There is a culture of emphasizing efficiency in mathematics," says Osvaldo Soto, who coaches math teachers as field director of the San Diego chapter of Math for America. "Math problems do not all take five minutes to solve." Not only does it lead students to disengage, but Soto says, it also reinforces the wrong habits needed for success. **Spending time** "mucking around," trying ideas, making mistakes, and then trying different ideas are paths to developing skills in deductive reasoning and making and supporting an argument. "That is one of the pillars of mathematics—the ability to reason from one point to another,"says Soto. Tricks and cutting to quick solutions cheat students of this learning (Pappano, 2014). Spending time with challenging math problems will be one approach to take a closer look at and how it is presented by teachers and executed by students in classrooms.

Teachers can tell students the correct answer and explain its corresponding rule, and most students will follow it, but this is good behavior, not mathematics learning or mathematical reasoning. Chances are that the rule will quickly be forgotten or, if remembered, will not be extended to other problem-solving situations. (Pogrow, 2004). As a teacher, it is not enough to present material with the answer, I argue that it goes deeper into their extended thinking that allow them to pose questions to one another that draws out their understanding, reasoning and making claims on how to solve the math problem.

Therefore, a second question that I plan to explore within the review of literature will be, 'To what degree will posing challenging math problems/tasks affect students' extended thinking?' In recent years, there has been a greater emphasis in research papers and curriculum documents on the important role played in problem solving by cognitively demanding tasks (Stein, Smith, Henningsen, & Silver, 2009). Also, most curriculum guidelines in mathematics education stress the need for teachers to **extend students' thinking**, and to pose extended, realistic and open-ended problems (City, Elmore, Fiarman, & Teitel, 2009) (Clarke, Roche, Cheeseman, van der Schans, 2014/2015).

Two projects with which we have been involved in recent years found that, on one hand, teachers seemed reluctant to pose challenging tasks to students and, on the other hand, students seemed to resist engaging with those tasks, and exerted both passive and active pressure on teachers to over-explain tasks or to pose simpler ones (Sullivan, Clarke, & Clarke, 2013) (Clarke, Roche, Cheeseman, van der Schans, 2014/2015).

Actions that result in a decline of the cognitive demand include routinising approaches to tasks, emphasis on completion rather than comprehension, inadequate time on the task, inappropriate choice of tasks, and expectations for high-level performance not being communicated. We would add concerns about over use of teacher modelling. By this we mean demonstrating to students how to solve the problem. Tzur (2008) argued that the two key times that teachers modify tasks are at the planning stage if they anticipate that students cannot engage with the tasks without considerable assistance, and once they see student responses if these are not as intended (Clarke, Roche, Cheeseman, van der Schans, 2014/2015). This statement is important to me as a teacher, as I want to be careful in my use of modelling to students so having students actively practice what I have modelled on similar problems will be a focus to address this concern.

Lastly, a third question I plan to discuss is, 'To what extent will challenging math problems/tasks impact students' persistence?' Challenging tasks are important for all students. Pogrow (1988) warned that by protecting the self-image of under-achieving students through giving them only "simple, dull material" (p. 84), teachers actually prevent them from developing self-confidence. He maintained that it is only through success on complex tasks that are valued by the students and teachers that such students can achieve confidence in their abilities. There will be an inevitable period of struggling while students begin to grapple with problems, but Pogrow asserted that this "controlled floundering" is essential for students to begin to think at higher levels (Clarke, Roche, Cheeseman, van der Schans, 2014/2015).

Ways Where Lack of Persistence Have Been Addressed

Use of Challenging Tasks: The Victorian Professional Learning Approach. My research first took me “down unda,” pardon the punt, in view of The Victorian Professional Learning Approach in Australia, see Appendix I. Each lesson has what we have come to call a **main task,** and this is often accompanied by an **introductory task** and **consolidating tasks**. An important feature of the documentation is the inclusion of enabling prompts (for students who have difficulty making a start on the main task) and extending prompts (for students who find the main task quite straightforward) (see Sullivan, 2011). To give a further sense of the kinds of tasks in these lessons, we include below the main task from two other lessons (see Figures 2 and 3). Work out which card is better value. Do this in two different ways. Explain your thinking clearly (Clarke, Roche, Cheeseman, van der Schans, 2014/2015).

The element within this use of a challenging math tasks in the form of tasks enabling a starting/**entry point** by way of an introductory task, main task and then consolidating task fit in with the common core standard of having entry points into a problem. As I progressed in my research, still down unda, I came accross, The Tasmanian Professional Learning Approach**.**

Use of Challenging Tasks: The Tasmanian Professional Learning Approach. A quite different approach professional learning was taken in Tasmania. Following the apparent success in using demonstration lessons to stimulate conversations around strategies for encouraging **persistence** in one school in Melbourne (see Cheeseman, Clarke, Roche, & Wilson, 2013), it was decided to use demonstration lessons as the main stimulus for Tasmanian teacher reflection on strategies that might prove helpful in encouraging students to **persist** with challenging mathematics tasks. (Clarke, Roche, Cheeseman, van der Schans, 2014/2015). Demonstration lessons, when situated within a professional development or coaching program, have been shown to hold the potential to promote teacher change and raise the quality of the teaching and learning in a classroom (Grierson & Gallagher, 2009; Joyce & Showers, 1980; Saphier & West, 2010).

- the presentation of theory within the professional learning program,
- professional support embedded in the workplace,
- the coach's or demonstration teacher's interpersonal skills and on-going support,
- structured feedback,
- the examination of evidence of student learning, collaborative planning and reflection on practices, with demonstration lessons or
- modelling being a key component (see, for example, Loucks Horsley, Love, Stiles, Mundry, & Hewson, 2003)(Clarke, Roche, Cheeseman, van der Schans, 2014/2015).

The element within this use of a challenging math tasks in the form of **modelling being a key component,** is an piece of intervention that I want to incorporate. However, where I would find immediately would be to change the roles where I would model to my students, give them feedback during collaboration practice. I realize that I am drawing closer to potential intervention structures of implementation.

Flipped Classroom. The popularity of **flipped classrooms** has been growing worldwide and developing nations have not been left behind. It is a model that rescheduled time spent in an outside classroom and its main strength is the shifting of learning ownership from the teacher to the student (Kashada, Abubaker, Hongguang, Chong Su, 2017) (Clarke, Roche, Cheeseman, van der Schans, 2014/2015).

Flipped classrooms in education are not describable as a new idea although the idea has been gaining plenty of popularity in math classrooms, middle schools and high schools [1]. A flipped classroom is a model of learning that readjusts and reschedules time spent both in and out of the classroom that enhances the shifting of ownership of learning to the students undertaking the studies from the teachers and educators [1]. Before attending class, students watch lecture videos, engaging their colleagues in the undertaking of project-based learning objectives, listening to podcasts and other remote instruction forms. After the out of class activities, the students are able to participate in attempts to solve challenges or problems designed to enhance content understanding. There are various benefits of making use of flipped classrooms including increased student engagement, an undertaking of meaningful homework and enhanced learning (Kashada, Abubaker, Hongguang, Chong Su, 2017) (Clarke, Roche, Cheeseman, van der Schans, 2014/2015). .

Although, the flipped classroom gives an impressive call to **time spent** both in and out of the classroom that enhances the shifting of ownership of learning to the students, it would not be attainable nor sustainable for me to create such a room within the time limits during the second semester of my classroom.

Extendend Thinking Framework. The framework is broken down into three student thinking strategies, see Appendix C:

- **Eliciting actions** allowed teachers to access students' existing thinking and to make it public. Eliciting actions provide students with opportunities to express their existing thinking about their mathematical activity or a mathematical phenomenon. They also allow teachers to become knowledgeable about their students' existing thinking so that they can use this information to decide which ideas to pursue (Cengiz, Kline, Grant, 2011).
- **Supporting actions** assisted students in remembering or visualizing what they already knew, and in considering new information. Supporting actions could be viewed as less desirable because they involve more teacher telling. However, the data indicate that supporting actions play a significant role in extending episodes. Students do not always make reasonable connections between their existing knowledge and new ideas, reason through their mathematical observations, or thoroughly articulate their thinking. It seems they need assistance doing these things in order to focus their mathematical reflection and reasoning on important mathematical concepts (Cengiz, Kline, Grant, 2011).
- **Extending actions** encouraged students to *move beyond* the initial mathematical activity. The extending actions are critical in creating opportunities for extending student thinking. After student thinking is elicited, teachers need to utilize instructional actions that allow students to further develop connections among ideas and solution methods and to move beyond their existing mathematical knowledge (Cengiz, Kline, Grant, 2011).

Summary

In recapitulation, I covered the definition of persistence and challenging math tasks, defrinciateing between persistence and motivation, what literature says in regards to three of my overaching questions and ways where lack of persistence have been addressed leading into my chosen intervention.

Many of my math students lack persistence in making sense of challenging math problems and persevere in solving them. They lack the persistence to extend their thinking by spending time trying ideas, making mistakes, applying strategies and reasoning deductively.

Within the Extended Thinking Framework, the extending action of 'move beyond' evokes the behavior of persistence that I seek for my students who lack persistence. Basing from the definition of persistence, stated earlier in this action research project, for which it is inclusive of allowing the possibility of **sustained thinking** through challenging tasks; I confidently decided that the the Extended Thinking Framework would respond to finding an an entry point to a problem, multiple attempts/spending time on problems, asking their peers for help or assisting others and academic conversations among students resulting from modeled teaching.

Theory of Action

If we teach students how to explain to themselves the meaning of a math problem by looking for entry points in to its solutions, provide reasoning for their claims, reminding each other of the goal of the discussion, the problem, or other information then students will persist by **extending their thinking** and **spending time** on challenging math problems/tasks.

Intervention

In narrowing my focus, by the end of this intervention, the shift that I want to see in student work into extended thinking will be…

- finding an an entry point to a problem
- multiple attempts/spending time on problems
- asking their peers for help or assisting others
- academic conversations among students resulting from modeled teaching

In an effort to counteract lack of persistence, the aim of this intervention will be to **sustain student thinking** using the 'extending student thinking framework': encouraging mathematical reflection, going beyond initial solution methods, and encouraging mathematical reasoning. This framework would take place in extending episodes. **This intervention** would be student-led instead of teacher-led in that student-led eliciting actions, extending actions and supporting actions between their peers provide opportunities to occur.

The intervention I chose to address students lack of persistence when faced with challenging math problems' was the *Extending Student Thinking Framework*. Although the research of the Extending Student Thinking Framework in Appendix D gave me data showing the impact of each mini strategy, my first thought was to choose all the mini strategies, however, this would not be sustainable for me as a teacher, and I would need more time than the 6 weeks for which I calendared this intervention to take place. I continued to review each mini strategy and its data showing which had the most impactful. After much consideration from the panel I presented the proposal of my problem of practice, I reconsidered to choose one strategy from each bucket. The most convincing point from the panel was that, my students were English Learners, and quality had more impact over quantity of strategies. Decreasing my intervention to one strategy per bucket and giving 2 weeks of intervention to two of the three strategies would allow for students to spend more time learning, practicing and applying the strategy rather than it becoming an overwhelming flow of strategies causing disengagement, confusion and lack of understanding.

The next question, I asked myself, which strategies should I choose out the several. The **Eliciting actions** bucket, was the easiest because there was only one strategy, 'Inviting students to share methods.' Eliciting actions provide students with opportunities to express their existing thinking about their mathematical activity or a mathematical phenomenon. They also allow teachers to become knowledgeable about their students' existing thinking so that they can use this information to decide which ideas to pursue… These actions included inviting students to share their solution methods by posing questions such as, ''How did you solve the problem?'' (Cengiz, Kline, Grant, 2011).

The next two mini-strategies took some time for me to choose. I knew that I would be choosing one mini-strategy from the bucket of Extending Actions and one mini-strategy from the bucket of Supporting Actions. My first inclination was to choose the strategy listed with the highest quantitative impacting data. When I read through the mini-strategies within **Extending actions**, it would seem likely to choose 'Evaluating a claim or an observation' because it had the highest overall frequency used in class 24. However, I chose the second highest impactful strategy for this section which was 'providing reasoning for a claim'.

The extending actions are critical in creating opportunities for extending student thinking. After student thinking is elicited, teachers need to utilize instructional actions that allow students to further develop connections among ideas and solution methods and to move beyond their existing mathematical knowledge.

Another prevalent extending action was inviting students to provide reasoning for their claims and solution methods by posing questions such as, What makes you say that? How do you know? Why do you suppose that? (Cengiz, Kline, Grant, 2011).

The Extending Student Thinking Framework listed a number of mini strategies under three buckets called Eliciting Actions, Extended Actions and Supporting Actions. Within those three buckets I finally choose one extending episode strategy from each bucket which were:

- Eliciting actions - Inviting students to share methods
- Extending actions - Inviting students to: provide reasoning for a claim
- Supporting actions - Reminding students of the goal, discussion or other information

The goal of my intervention towards addressing students' lack of persistence when faced with challenging math problems is to give students an entry point into challenging math problems, ask questions to peers and spend more time with challenging math problems. In addition, I aim to answer the following research questions:

➢ Do students 'invite their peers to share methods' by posing questions?

➢ Do students 'provide reasoning for a claim' by stating a claim and posing questions?

➢ Do students "remind each other of the goal of the discussion, the problem, or other information" by posing questions?

Defining Extending Episodes. What are extending episodes? ...a segment of a whole-group discussion that focused on a particular mathematical idea or solution, and that involved mathematical reflection or reasoning or going beyond initial solution methods, was conceptualized as an extending episode (Cengiz, Kline, Grant, 2011). In view of Appendix C, 'Table 1 Extending student thinking framework' found in the article, 'Extending students' mathematical thinking during whole-group discussions' (Cengiz, Kline, Grant, 2011), I plan to execute the following intervention.

The role of extending episodes: eliciting, supporting, and extending actions. Extending episodes usually began right after a mathematical idea or solution(s) were shared and ended when the mathematical focus of the discussion changed (Cengiz, Kline, Grant, 2011).

The Role of Eliciting Actions. Eliciting actions provide students with opportunities to express their existing thinking about their mathematical activity or a mathematical phenomenon. They also allow teachers to become knowledgeable about their students' existing thinking so that they can use this information to decide which ideas to pursue (Cengiz, Kline, Grant, 2011).

These actions included inviting students to share their solution methods by posing questions such as, ‘‘How did you solve the problem?’’The most commonly observed extending action was inviting students to evaluate a claim or an observation (Cengiz, Kline, Grant, 2011).

Teachers encouraged students to reflect on their shared ideas and solution methods by usually asking questions such as, What do you think? Do you agree? Do you think it's true? This action provided students with an opportunity to collectively reflect on their shared mathematical ideas or solution methods, and further recognize connections between their existing knowledge and their observations. It also allowed teachers to become aware of what their students were thinking about particular mathematical issues (Cengiz, Kline, Grant, 2011).

The Role of Extending Actions. The extending actions are critical in creating opportunities for extending student thinking. After student thinking is elicited, teachers need to utilize instructional actions that allow students to further develop connections among ideas and solution methods and to move beyond their existing mathematical knowledge. Another prevalent extending action was inviting students to provide reasoning for their claims and solution methods by posing questions such as,

- What makes you say that?
- How do you know?
- Why do you suppose that?

The Role of Supporting Actions. Supporting actions could be viewed as less desirable because they involve more teacher telling. However, the data indicate that supporting actions play a significant role in extending episodes. Students do not always make reasonable connections between their existing knowledge and new ideas, reason through their mathematical observations, or thoroughly articulate their thinking. It seems they need assistance doing these things in order to focus their mathematical reflection and reasoning on important mathematical concepts (Cengiz, Kline, Grant, 2011).

The teacher sometimes sharing their own interpretations of their observations and of students’ claims are supporting actions. Reminding students of information that is related to the problem they were solving or to the ideas they were discussing was another supporting action that helped students form connections between their new observations and what they already knew (Cengiz, Kline, Grant, 2011).

Repeating students’ claims or having students repeat each other’s claims was also useful for having students engage in discussions or stay focused on ideas being discussed. Another instructional action that supported the extending episodes was recording student thinking on the board. Having a recording of shared ideas or solution methods allowed students to collectively reflect on their thinking (Cengiz, Kline, Grant, 2011).

In order to answer the Extended Thinking Strategy Questions listed below,

- ➢ Do students ‘invite their peers to share methods’ by posing questions?
- ➢ Do students ‘provide reasoning for a claim’ by stating a claim and posing questions?
- ➢ Do students “remind each other of the goal of the discussion, the problem, or other information” by posing questions?

I decided to create a general template of the strategies questions that could be applied to any math problem as well as outside of my classroom. Creating a tool for the students to use with them academically, transferable and personally was a crucial part in my daily lesson planning. I felt that it would allow them to find it useful in multiple areas of their lives.

Data Collection and Analysis & Findings

Research Methods. To collect data, I created a data collection spreadsheet (hyperlink) or see Appendices E-H, in order to:

- ❏ Track the number of students asking the strategic questions that I choose from the Eliciting Actions, Extending Actions and Supporting Actions for the intervention. I analyzed the increase and/or decrease of number of students asking the strategic questions from the Eliciting Actions, Extending Actions and Supporting Actions intervention.
- ❏ Track the questions and comments that were happening outside of the protocol strategy questions. I analyzed the amount of questions/comments and quality of questions/comments that students were engaging in to indicate or not indicate prolonged conversations about the challenging math problems.

Observationally, I spent 2-minute intervals with each pair of Student A & B in order to hear students asking questions to their peers and using my modelled responses to the strategy questions as I notated on my data collection spreadsheet. I analyzed this data by comparing any correlations, patterns and/or anomalies from the number of students of whom I heard asking the protocol questions, questions/comments outside the strategy protocol as I walked around the room to that of the data from the students' Exit Ticket surveys and also to that of the data collection spreadsheet. In order to capture the students' work, I collected:

- ❏ Daily Exit Tickets where students recorded their responses to the the strategy questions to allow me to analyse their work as pairs and as a whole class. I analyzed the students' daily exit ticket survey responses in comparison to my observations recorded on my data collection spreadsheet in search of similarities, differences or anomalies.

❏ Hard copies of student work in solving the challenging math problem. I analyzed the students' hard copy to see if students were persisting through the math problem using the strategic questions by the show of the quantity and quality of their written work on the challenging math problem.

Plan of INTERVENTION	**Component**	**Activities**	**Purpose / Sub-Question to be answered**	**Data to be Collected**	**Type of Data (process vs. impact)**
Week 1 (1 day)	Pre-Student Intervention Survey (self-monitoring) as Exit Ticket.	Student Extended Thinking Pre-Survey (4/6/18)	Which statements do students identify with when they are faced with challenging math problems.	Baseline	Impact
Week 2 (1 week of practice)	Extending Student Thinking Framework: **Eliciting actions**	1) Teaching lessons on "Inviting your peers to share methods" • Posing questions such as, ''How did you solve the problem?'' 2) Exit Tickets • 1st Strategy Daily Exit Ticket 4/9/18 • 1st Strategy Daily Exit Ticket 4/12/18 • 1st Strategy Friday Exit Ticket 4/13/18	1) Do students 'invite their peers to share methods' by posing questions such as: • "How did you solve the problem?" • Follow-up question 1: "What solution method did you use?' • Follow-up question 2: "Why did you use that solution method?" • Follow-up question 3: "How did you know to use that solution method?" 2) How many students used the questions?	Daily Exit Ticket asking, "Rate how effective today's strategy on 'Inviting my peers to share their methods" allowed you to have longer conversations about the math problem." Additional question on Friday Exit Tickets "Explain how or how not this strategy has worked for you?"	Qualitative Process
Week 3 & 4 (2 weeks of practice)	Extending Student Thinking Framework: **Extending actions**	1) Teaching lessons on "Inviting students to: Provide reasoning for a claim. • Make a claim • What makes you say that? • How do you know? • Why do you suppose that?	1) Do students 'provide reasoning for a claim' by stating a claim and posing questions such as: • Make a claim by choosing and stating the best method for solving the problem • What makes you say	Daily Exit Ticket asking, "Rate how effective today's strategy on 'provide reasoning for a claim' allowed you to have longer conversations	Qualitative Process

		2) Exit Tickets • 2nd Strategy Daily Exit Ticket 4/16/18 • 2nd Strategy Daily Exit Ticket 4/19/18 • 2nd Strategy Friday Exit Ticket 4/20/18 • 2nd Strategy Daily Exit Ticket 4/23/18 • 2nd Strategy Friday Exit Ticket 4/26/18 • 2nd Strategy Friday Exit Ticket 4/27/18	that? • How do you know? • Why do you suppose that? 2) How many students used the questions?	about the math problem." Additional question on Friday Exit Tickets "Explain how or how not this strategy has worked for you?"	
Week 4 & 5 (2 weeks of practice)	Extending Student Thinking Framework: **Supporting actions**	1) Teaching lessons on "Students reminding each other of the goal of the discussion, the problem, or other information" 2) Exit Tickets • 3rd Strategy Daily Exit Ticket 4/30/18 • 3rd Strategy Daily Exit Ticket 5/3/18 • 3rd Strategy Friday Exit Ticket 5/4/18 • 3rd Strategy Daily Exit Ticket 5/7/18 • 3rd Strategy Friday Exit Ticket 5/11/18	Do students "remind each other of the goal of the discussion, the problem, or other information," by posing questions such as: • "What do you already know, such as the given information in the problem?" • "What is the question asking?" • What do we need to solve for? • Follow-up question 1: "What method did you use to solve?" • Follow-up question 2: "How did you solve the problem?" 2) How many students used the questions?	"Rate how effective today's strategy on 'reminding each other of the goal of the discussion, the problem, or other information" allowed you to have longer conversations about the math problem." Additional question on Friday Exit Tickets "Explain how or how not this strategy has worked for you?"	Qualitative Process
Week 6 (1 week of students using ALL strategies practice)	**Observing:** Extending Student Thinking Framework: Eliciting actions Extending actions Supporting actions	1) Teach lessons using the three strategies taught: • 1st Strategy: "Inviting your peers to share methods" • 2nd Strategy: "Inviting students to: • Provide reasoning for a claim. • 3rd Strategy: "Students reminding each other of the goal of the discussion, the problem, or other information"	1) Do students 'invite their peers to share methods' by posing questions such as: • "How did you solve the problem?" • Follow-up question 1: "What solution method did you use?' • Follow-up question 2: "Why did you use that solution method?" • Follow-up question 3: "How did you know to use that solution method?" Do students 'provide	Daily Exit Ticket asking, "Rate how effective today's strategies on 'Inviting my peers to share their methods" allowed you to have longer conversations about the math problem, Provide reasoning for a claim and Reminding each other of the goal of the discussion,	Qualitative Process

		2) Exit Tickets • All-Strategies Daily Exit Ticket 5/14/18 • All-Strategies Daily Exit Ticket 5/17/18 • All-Strategies Friday Exit Ticket 5/18/18	reasoning for a claim' by stating a claim and posing questions such as: • Make a claim by choosing and stating the best method for solving the problem • What makes you say that? • How do you know? • Why do you suppose that? Do students "remind each other of the goal of the discussion, the problem, or other information," by posing questions such as: • "What do you already know, such as the given information in the problem?" • "What is the question asking?" • What do we need to solve for? • Follow-up question 1: "What method did you use to solve?" • Follow-up question 2: "How did you solve the problem?" 2) How many students used the questions?	the problem, or other information " allowed you to have longer conversations about the math problem." Additional question on Friday Exit Tickets "Explain how or how not all strategies have worked for you?"	
Week 7 (1 day)	Post-Student Intervention Survey/Rubric (self-monitoring)	1) Student Extended Thinking Post-Survey 2) I will observe and collect data on strategies being used among students on the first day of the first week and the last day of the last week during the intervention.	Did students use any of the strategies listed for extended thinking? Which strategies did the students identify as using? What were impacts and learnings from the findings and analysis of the exit ticket and observational data.	After Intervention Findings & Analysis	Impact

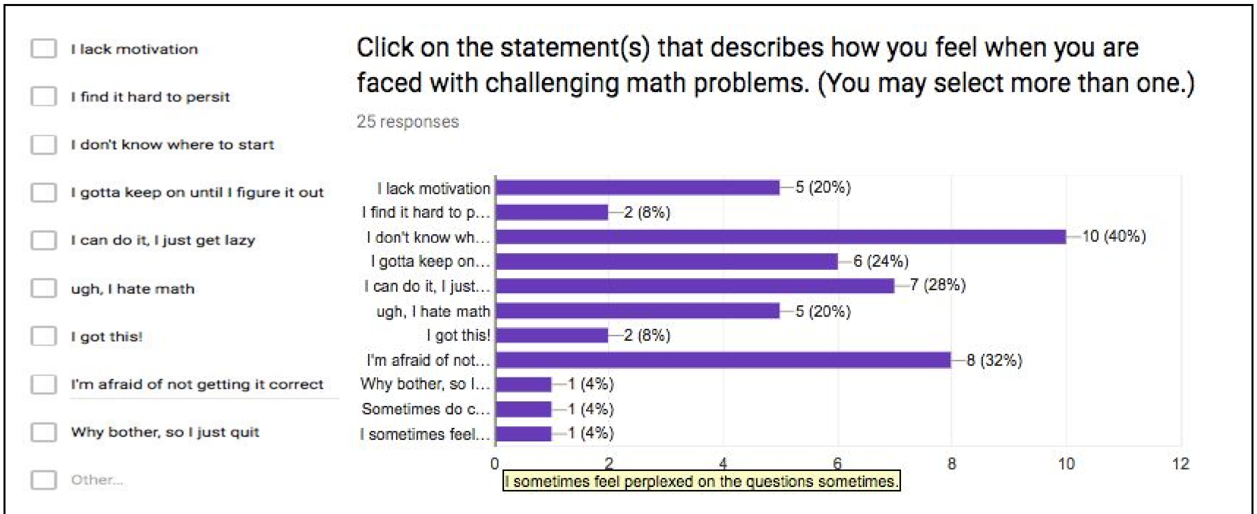
Pre & Post Survey Overall Findings. I looked at the contrast of the pre and post survey student choosen statements to discover positive and negative shifts in their perspectives when faced with challenging math problems. I reviewed studeents' chosen statements to see which choice of statements had increased and decreased. From this analysis I was able to determine the positive and negative shifts in their view when faced with challenging math problems.

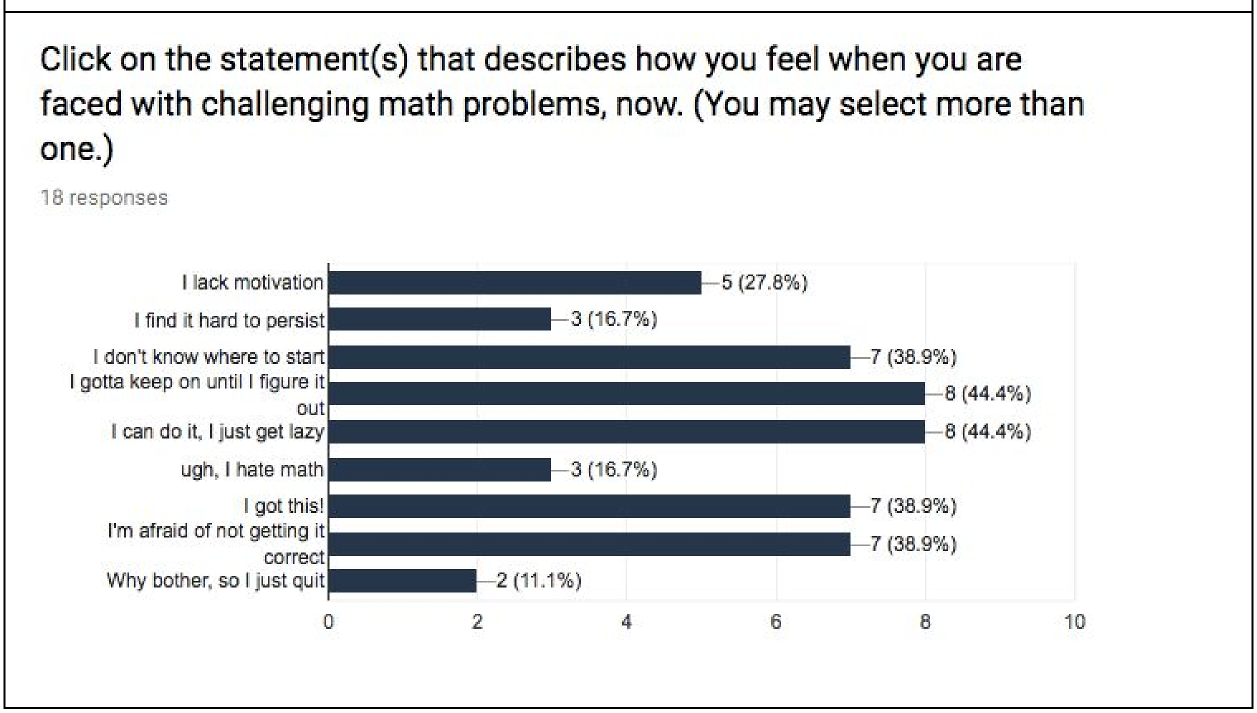
In comparison of my pre & post survey, I found that my students had an overall change from a negative to subtle positive shift when faced with challenging math problems. In my baseline survey, students were to 'Click on the statement(s) that describes how you feel when you are faced with challenging math problems. (You may select more than one.)' Initially there were 10 students who chose "I don't know where to start", and in the post survey only 7 students chose this statement. This revealed a subtle decrease of 1.1% student negative view to a positive view.

Another example of a positive view shift, was seen in the data that in the pre survey, 6 students chose, "I gotta keep on until I figure it out.", whereas, in the post survey 8 students choose this statement; a 20.4% increase shift towards a positive view.'

The most impactful finding was that of pre-survey data showcasing only 2 students chose the statement, "I got this". Subsequently, after the intervention, it increased to 7 students choosing this statement. This was a 34.9% increase in confidence.

Consequently one notable negative finding is seen where in the pre-survey, only 1 student chose the statement of "Why bother, so I just quit", whereas 2 students chose this statement in the post survey. Both of those students were repeating my math class. It is possible that their mindset had been affected by previously failing math class. It is also possible that they both needed more practice in using the strategy questions instead of only practicing during my 7-week intervention.





Changes to Intervention & Data collection. During my first week, I administered the front page of 'Teacher Models' the strategy and on the back side would be the "Student Practice" similar problem but with different numbers. I noticed that while I collected the student work, students were not able to have a copy of the Teacher Model side. So, I separated the sheet into two separate sheets. This way students were able to have a their own copy of what was modeled that day and it serves as a resource for the students to use later on for the same topic and for using this strategy. I made this change starting from the second week until the end of the Intervention.

For future use of my intervention's three strategies, what I would do differently would be to introduce the strategies in this particular chronological order:

1. 3RD STRATEGY: Supporting actions
2. 2ND STRATEGY: Extending Actions
3. 1ST STRATEGY: Eliciting actions

I would start with the 3rd strategy (Supporting Actions) first, which was:

3RD STRATEGY: **Supporting actions**

- ❏ **What do you already know, such as the given information in the problem?**
- ❏ **What is the question asking?**
- ❏ **What do we need to solve for?**

FOLLOW-UP QUESTIONS:

- ❏ *How did you know to use this method to solve?*
- ❏ *How did you solve the problem?*

The reasoning behind starting with this strategy first was that I found it to be the essential starting point when approaching any math problem or any other subject. When a student poses the question, “What do you already know, such as the given information in the problem?’ allowed the students to easily identify simple pieces of the challenging math problem as opposed to the the 1st strategy that required them to express “How they solved the problem?” In addition, I would not add the follow-up questions because, I felt that those questions were redundant to the first strategy under Eliciting Actions, which calls for students to invite each other to share their solution methods, seen below. Furthermore, after the student makes a claim from the second strategy this would lead them to solve the problem therefore being able to explain how they solved the problem.

1ST STRATEGY: **Eliciting actions**

- ❑ **‘‘How did you solve the problem?”**

FOLLOW UP QUESTIONS:

- ❑ *What method did you use?*
- ❑ *Why did you use that method?*
- ❑ *How did you know to use that method?*

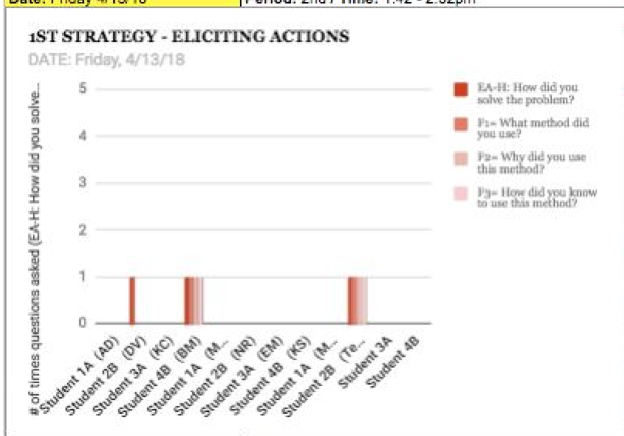
Process of data analysis. Basically, I collected the student work each day and tallied the observations on the data excel sheet. I tracked their exit reponses to see how they transposed the modeled example to their own words for the the similar problem of that day. Students’ responses were significantly similar to my own words as they transposed their responses to fit their challenging math problem which was different to my challenging math problem by way of integers used.

Observationally, I captured additional peer-to-peer questions outside of the strategy protocol questions. I notated the number of questions and the actual questions they were asking and how it changed over time. Students used the strategy model but they increased in asking other questions which allowed for longer conversations in spending time on the problem.
I reviewed all exit ticket responses and in the particular, the last day’s exit ticket because it included all three strategies along with the student ratings of the strategies used and its contribution to their learning. I analyzed the students’ wording in their responses to see how they incorporated modeled wording within their own.

Perseverance Intervention-Impact and Learnings. An overarching question that I asked myself was, ‘Will this intervention increase students’ perseverance when solving challenging math problems? Based on my learnings, it is an emphatic yes!
During this Intervention process, the change impact-wise, was that students were **spending more time** by asking more questions outside of the structured strategy questions provided for them to follow and they were using my wording to incorporate into their own responses in the exit tickets.
The time being spent by students was seen in two ways:

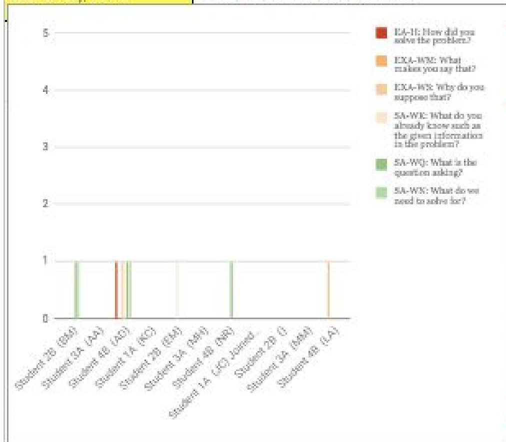
- the increase of the students <u>using the structure of strategic questions</u>
- which then expanded into them asking more questions/comments on their own

After analyzing my data that I collected at the end of the first week; it showed that there were no additional student questions asked; they strictly followed the protocol of questions seen below.

Date: Friday 4/13/18	Period: 2nd / Time: 1:42 - 2:32pm	Eliciting Actions STRATEGY: Students invite each other to share their solution methods.				Student questions asked or comments made.
		EA-H: How did you solve the problem?	F1= What method did you use?	F2= Why did you use this method?	F3= How did you know to use this method?	
	Student 1A (AD)					
	Student 2B (DV)	1				
	Student 3A (KC)					
Stanford	Student 4B (BM)	1	1	1	1	
	Student 1A (MH)					
	Student 2B (NR)					
	Student 3A (EM)					
Dickinson College	Student 4B (KS)					
	Student 1A (MM)					
	Student 2B (Teacher)	1	1	1	1	
	Student 3A					
UC Berkely	Student 4B					

However, by the last week of the intervention featuring all three strategies used, my analysis of my data collected revealed that there were 12 additional questions/comments seen the the following two screenshots. My data also affirms that **perseverance** had increased due to the students **spending time** in asking other questions and/or making comments during the solving of the challenging math problem.

Date: Monday, 5/14/18 | **Period/Time:** 2nd Period / 9:23-10:15am



		Eliciting Actions	Extending Actions			Supporting Actions			
		STRATEGY: Students invite each other to spafe their solution methods.	STRATEGY: Students invite each other to provide reasoning for claims/solution methods.			STRATEGY: Students remind each other of the goal of the discussion, the problem , or other information			Student questions asked or comments made.
		EA-H: How did you solve the problem?	**EXA-WM:** What makes you say that?	**EXA-H:** How do you know?	**EXA-WS:** Why do you suppose that?	**SA-WK:** What do you already know such as the given information in the problem?	**SA-WQ:** What is the question asking?	**SA-WN:** What do we need to solve for?	
	Student 1A (DV)								Where did you get 9 from?
	Student 2B (BM)						1	1	
	Student 3A (AA)								She squared it, that is how she(Teacher) got 9 and then she subtracted it. (Note: Student walked over to DV and showed him how he solved for the vertex.)
Harvard	Student 4B (AD)	1			1	1	1	1	
	Student 1A (KC)								How do you know how to do that? Oh you...What is this for? That's the vertex? How did you get the vertex out of -2? Where did you get the -2 from?
	Student 2B (EM)					1			It's the vertex. The k is like a y. We solved one (pair of points) now we need to find the other two(pair of points)
	Student 3A (MH)								
Dickinson College	Student 4B (NR)						1		I dont get how we got a -3 (vertex in Teacher model. Note: Teacher assits studemnt to understand)
	Student 1A (JC) Joined NR & MH								(Note: Student walked over to DV to confirm if he solved for the vertex correctly. DV confirmed that he did)
	Student 2B ()								
	Student 3A (MM)								
UC Berkely	Student 4B (LA)		1						What's your claim?

Amidst unpacking my data analysis, I notated significant conversations transpiring among the Student A/B pairs. Below, here are questions and comments that Student A/B pairs were having with each other for which I observed and quickly notated on my data collection tool.

Student questions asked or comments made.
Where did you get 9 from?
She squared it, that is how she(Teacher) got 9 and then she subtracted it. (Note: Student walked over to DV and showed him how he solved for the vertex.)
How do you know how to do that? Oh you...What is this for? That's the vertex? How did you

get the vertex out of -2? Where did you get the -2 from?
It's the vertex. The k is like a y. We solved one (pair of points) now we need to find the other two(pair of points)
I dont get how we got a -3 (vertex in Teacher model. Note: Teacher assist student to understand)
(Note: Student walked over to DV to confirm if he solved for the vertex correctly. DV confirmed that he did)
What's your claim?

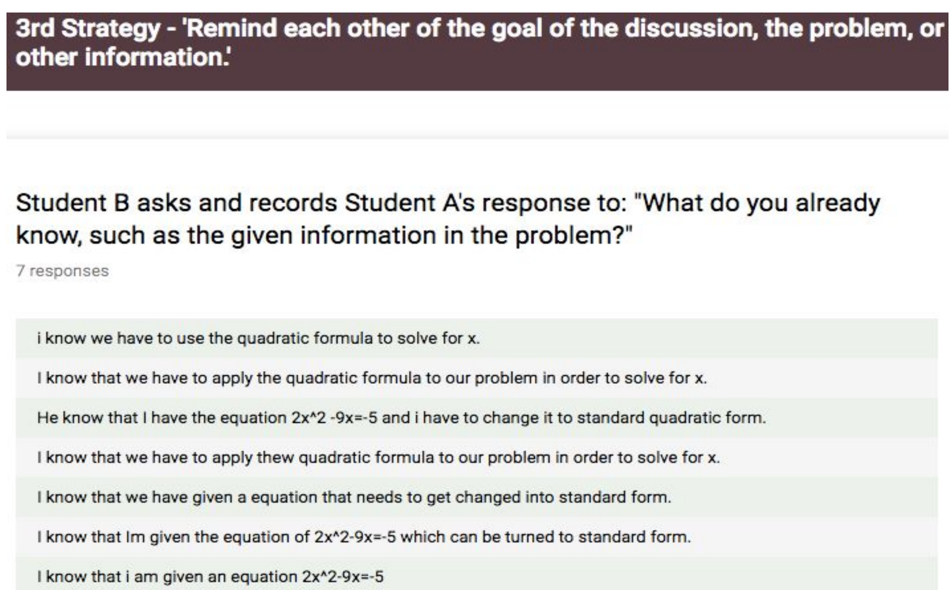
The connection from this data to the increase in perseverance was that students were given a strategy of questions to ask when faced with a challenging math problem while problem solving and from this produced peer-to-peer additional questions and comments outside of their protocol, therefore increasing the amount of time that they were **spending on the challenging math problem.**

Perseverance Intervention Process Data - Exit Tickets. One of the first process data after this intervention was that of modeling the strategies to students. Modeling the strategies to students allowed students to practice incorporating these strategic questions when faced with different challenging math problem.

A second process data is that students used my words from the modeling in other context of challenging math problems. I observed students using my modelled wording in their answers as it was being verbally expressed and recorded between Student A & B. In addition, data supported my observations through the analysis of their exit ticket reponses.

A third process data within my findings were that of having clear tools of the strategies opened the door for academic conversation that included the use of academic vocabulary and staying longer in the process of the problem solving itself. Here are samples of my modelled notes and student exit ticket responses on their last day of the intervention.

In my first sample with my modeled response for answering the 3rd strategy question, "What do you already know, such as the given information in the problem?" I answered saying, "I know that I am given an equation $4x^2 = 8x - 1$ that I can change into standard quadratic form." The Student B responses varied but followed the same structure of my response when faced with a similar challenging math problem only different numbers.

Teacher Sample:	Students' B responses on exit ticket
➢ **3RD STRATEGY: Supporting actions** **Student 1: What do you already know, such as the given information in the problem?** **Teacher:** I know that I am given an equation $4x^2 = 8x - 1$ that I can change into standard quadratic form. **Student 2: What is the question asking?** **Teacher:** The question is asking to use	 **3rd Strategy - 'Remind each other of the goal of the discussion, the problem, or other information.'** Student B asks and records Student A's response to: "What do you already know, such as the given information in the problem?" 7 responses i know we have to use the quadratic formula to solve for x. I know that we have to apply the quadratic formula to our problem in order to solve for x. He know that I have the equation 2x^2 -9x=-5 and i have to change it to standard quadratic form. I know that we have to apply thew quadratic formula to our problem in order to solve for x. I know that we have given a equation that needs to get changed into standard form. I know that Im given the equation of 2x^2-9x=-5 which can be turned to standard form. I know that i am given an equation 2x^2-9x=-5

the quadratic formula. **Student 3: What do we need to solve for?** **Teacher:** I need to solve for x. I must round my answer to the nearest hundredth. If there is more than one solution, separate them with commas.	

A second sample of students using my modeled words featured in the 2nd strategy, is when I make a claim: "The best method to use will be the solutions to the quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$". Students' claims were very similar with the exception of writing the actual formula. Here you see some claims written in transcript due to the inaccessibility of not writing it in the equation formula form.

Teacher Sample:	Students' B responses on exit ticket
➢ **2ND STRATEGY**: **Extending actions** **Teacher** CLAIM: The best method to use will be the solutions to the quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ **Student 4: What makes you say that?** **Teacher:** I say that because you change the given equation $4x^2 = 8x - 1$ to standard form which is $4x^2 - 8x + 1 = 0$ **Student 5: How do you know?** **Teacher:** I know because that way you can identify $a = 4,\ b = -8$ *and* $c = 1$. **Student 6: Why do you suppose that?** **Teacher:** I suppose because you can plug in those values of a, b and c in the quadratic formula to solve for x.	**2nd Strategy - 'Inviting my peers to provide reasoning for claims/solution methods.'** What was Student A's claim? Type it here: 7 responses My claim is to use the quadratic formula. I claim the best method to use to solve the problem is the quadratic formula. The best method to use is the quadratic formula x=-b+- square root of b squared -4ac all over 2a The best method to use is the quadratic formula. to use the quadratic formula. best method to use is the square root method. the best method to use is the solutions to the quadratic formula

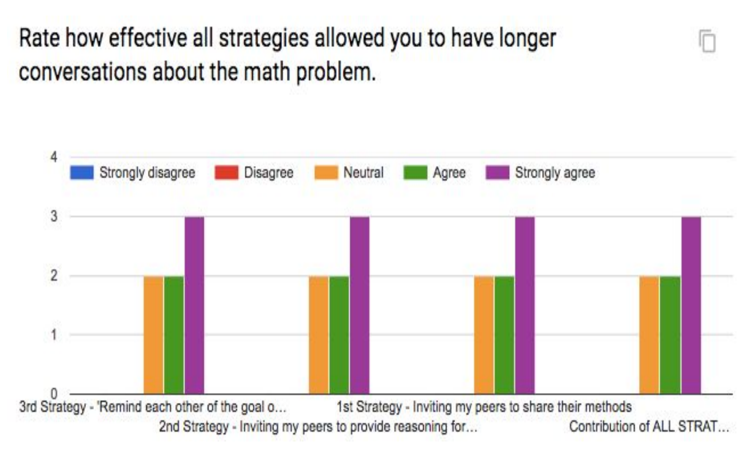
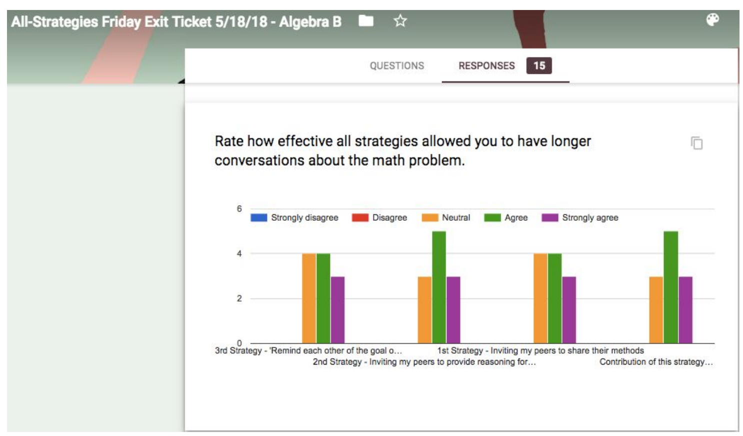
The 3rd strategy and 2nd strategy had 7 student exit ticket responses and were detailed. They spent their time to ask each other the strategy questions, solve the problem and respond with their thinking with the use of my modeled examples and clear tools.

A third sample of my students using my modeled words featured in the 1st strategy for **"How did you solve the problem?"**. My lengthy modeled response below to the right, was interpreted in their own words by 3 students. I observed that students were **having longer conversations** as I walked around notating comments and questions (showcased earlier) during students' exit ticket responses.

Teacher Sample:	Students' B responses on exit ticket
➢ **1ST STRATEGY: Eliciting actions** **Student 7: "How did you solve the problem?"** **Teacher:** I first rewrote the given equation in standard quadratic form as $4x^2 - 8x + 1 = 0$. Then I identified that $a = 4,\ b = -8$ *and* $c = 1$. Next, I plugged those values into the quadratic formula and got my solutions rounded to the nearest hundredths for $x = -0.13, -1.87$	**1st Strategy - 'Inviting my peers to share their methods'** Student B asks and records Student A's response to: "How did you solve the problem?" 3 responses i solve by figuring out the formula and i use ms,stephen method to solve the answer. we wrote the equation into standard form then i plugged in the values and rounded my solution to the nearest hundredths. Byusing the quadratic formula

The ending of the exit ticket asked the students to "Rate how effective all strategies allowed you to have longer conversations about the math problem". Below, Student A's ratings revealed no *strongly disagree* or *disagrees*. The category for *neutral* and *agree* were tied and the highest rating was *strongly agree*. This goes to my point that they were spending time which allowed for longer conversations.

Among the Student B's responses there were same ratings on no *strongly disagree* or *disagrees*. The category for *neutral* and *agree* were tied only for the 1st strategy and 3rd strategy but the 2nd strategy had the highest rating in the category of *agree*.

Student A Responses:	Student B Responses:
	

Perseverance Intervention Process Data: Spending time

The Eliciting, Extending and Supporting Actions of my intervention lead to students spending time and talking about the challenging math problem through the routine of modeling the strategy questions such as:

➢ Do students 'invite their peers to share methods' by posing questions?

- ➢ Do students 'provide reasoning for a claim' by stating a claim and posing questions?
- ➢ Do students "remind each other of the goal of the discussion, the problem, or other information" by posing questions?

The data I collected and mentioned earlier from my data spreadsheet and that of last day showed an increase in student asking questions outside of their protocol strategy questions. This is a clear indication of students **persisting** by spending more time and talking about the challenging math problem.

Conclusion

In review of the success or failure of the Extended Thinking Framework intervention for this action research, I conclude that it was not a failure yet there were subtle successful improvements in persistency when my students were faced with challenging math problems. Of the three strategies that I used, the most effective strategy was the use of all three strategies at once. My data indicates an increase in conversation with the use of the question starters along with students asking non-protocol academic questions and making comments therefore increasing in spending time on the challenging math problem.

Implications

Overall Takeaways for Teachers. As a result of this intervention, some overall takeaways for teachers in terms of increased student persistence was seen through conversation questions-starter strategies. The question-starter strategies increase conversations among students within peer-to-peer groups and allowing students to initiate conversation with other students outside their peer groups to seek other methods. It will allow for students to verbally express their metacognition amongst themselves. Students hear themselves formulate their own understanding which allowed them to engage metacognitively. I do believe that it allowed them to become unstuck and that the 'Extended Strategies' are transferable strategies to other content classes and for individual students be themselves or self-directed learners.

General Recommendations for Teachers. A cautionary limitation is that when using these strategical questions in the Extending Actions such as, "Make a Claim," it would be important for the teacher to model the claim by saying: "The best claim…" this way, students can have an opportunity to give an alternative claim. If the student starts their claim with just a method that they claim to use, it would not make the claim effective for it to be arguable. A good claim is arguable.

When using the strategic question in Supporting Actions such as, "How did you solve the problem?", a teacher should also provide follow-up questions such as:

1. What method did you use?
2. Why did you use this method?
3. How did you know to use this method?

This will allow for the development of conversations towards answer fleshing out the primary questions of this particular strategic question.

As a result of this intervention, I plan on continuing to incorporate those strategic questions from this intervention and the rest of the questions throughout the next academic year. I would spend more weeks on each question while interweaving it into the lesson and of course with modeling how to use and respond to the strategy questions. However, I would want to get to the point where students are automatically using the strategies without it being the protocol of the lesson. This will take time as literature research stated.

In my own newly practitioner's opinion, lack of persistence among students when faced with challenging math problems is a significant problem of practice and I do believe that I have only scratched the surface and that my problem of practice is a common occurrence in plethora of classrooms. However, I believe there is still a lot more digging to uncover where the source of lack of persistence derive. It is clear, that a starting point with question-starter strategies **increase student thinking** that then addresses breaking down the of lack of persistence when students are faced with challenging math problems causing them to **spend more time** with the challenging math problem therefore counteracting some measure of lack of persistence. But, ***thinking*** is the a crucial yet vital ingredient.

In the same vein of **sustained thinking** drawing from the definintion of persistence, this upcoming academic year, I am looking to explore the student 'thinking' in the case of word problems. I would want to confront the strong emotional impact that occurs when a student realizes that he or she does not know how to start but would want to shift/guide those immediate reactions of uncertainty to showing and using different ways to think through a word problem.

Bibliography

Bonnett, V., Yuill, N., Carr, A., Educational & Child Psychology, March 2017, Vol. 34 Issue 1, 83-93

Cengiz, N., Kline, K., Grant, T., Extending students' mathematical thinking during whole-group discussions, March 2011, 356-374

Clarke, D., Roche, A., Cheeseman, J., van der Schans, S., Mathematics Teacher Education & Development , 2014/2015, Vol. 16 Issue 2, 46-70

Cone, M., & Kenda, A., Transitioning to the Common Core in California Charter Schools: Challenges and Solutions, July 2015, 1-25

Martin, A., Using Load Reduction Instruction (LRI) to boost motivation and engagement, 2016, 1-60

Pappano, L.,. Education Digest, Changing the Face of Math, February 2014, Vol. 79 Issue 6, 10-14

Pogrow, S., Supermath: An Alternative Approach to Improving Math Performance In Grades 4 Through 9, December 2004, Vol. 86 Issue 4, 297-303

Sinicrope, R., Eppler, M., Preston, R., Ironsmith, M., School Science & Mathematics; Preservice Teachers of High School Mathematics: Success, Failure, and Persistence in the Face of Mathematical Challenges. February 2015, Vol. 115 Issue 2, p 56-65

Tulis, M., Fulmer, S., Learning & Individual Differences, October 2013, Vol. 27, p 35-46

Appendix A

The Motivation and Engagement Wheel from A.J. Martin (2016)

Andrew J. Martin

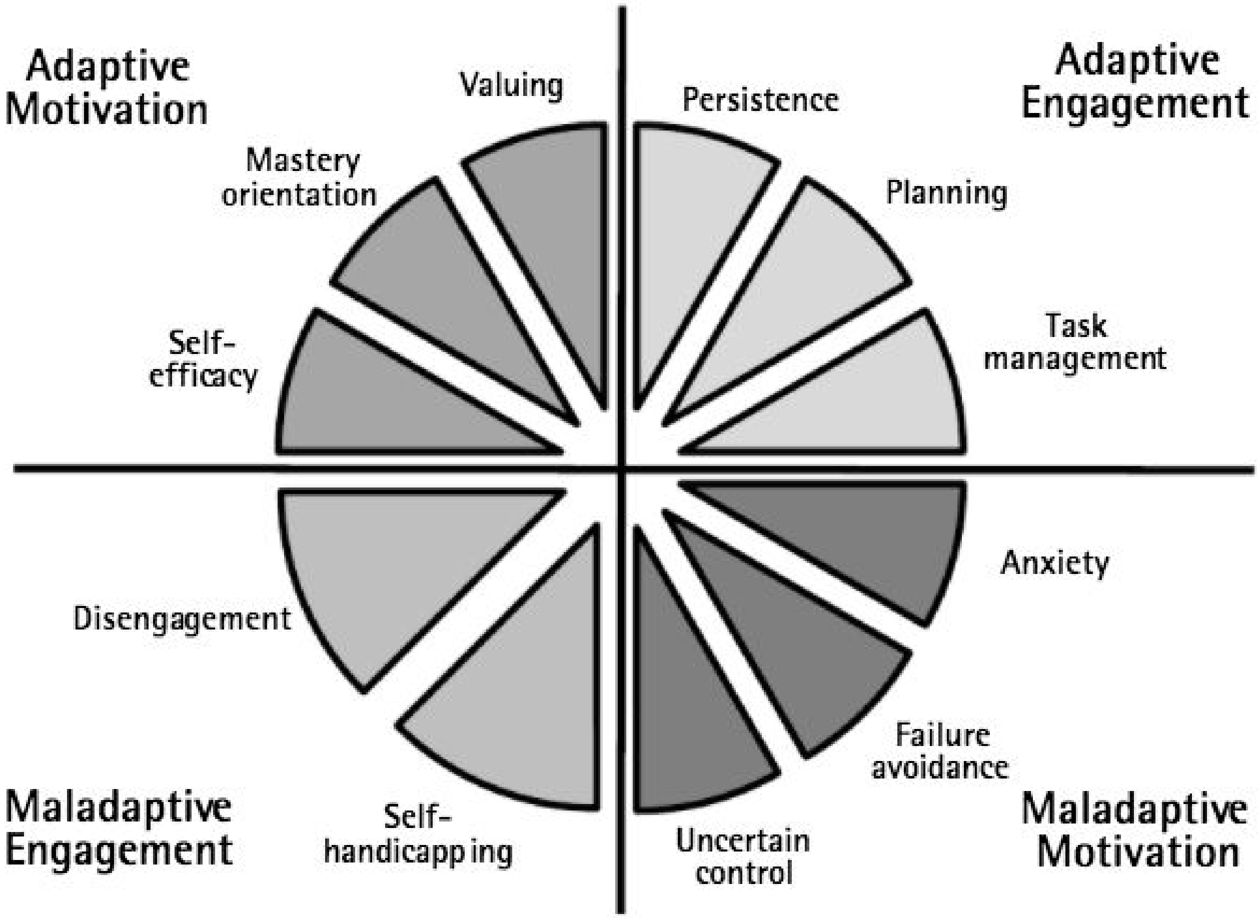


Figure 3: The Motivation and Engagement Wheel. Reproduced with permission from A.J. Martin and Lifelong Achievement Group (www.lifelongachievement.com).

Appendix B

Mr. M's Math Department Pre-diagnostic assessment, Unity's Basic 25.

* Unity's Basic 25 * Assessment ID: dna.15486 ib.1077078

Directions: Answer the following question(s).

1 $-4-(-2)=$

A. -2
B. -6
C. 2
D. 6

2 $5-(-2)=$

A. -3
B. -7
C. 3
D. 7

3 $27 \div (-9)=$

A. 3
B. -3
C. $\frac{1}{3}$
D. $-\frac{1}{3}$

4 A coin contains 1.25 grams of nickel and 3.75 grams of copper, for a total eight of 5 grams. What percent of the coins total weight is copper?

A. 40 %
B. 4 %
C. 25 %
D. 75 %
E. 2.5 %
F. 7.5 %

5 What number is 40% of 16?

A. .64
B. 6.4
C. 64
D. 4.0
E. 40
F. 400

6 16 is 40% of what number?

A. .64
B. 6.4
C. 64
D. 4.0
E. 40
F. 400

7 Mark bought two items in the store for $3.35 and $4.15. Sales tax is 8%.

What is the total cost of his purchases, including tax?

A. $13.5
B. $7.56
C. $8.10
D. $0.60

* Unity's Basic 25 * Assessment ID: dna.15486 ib.1077078

Directions: Answer the following question(s).

8 $8 \cdot 8 \cdot 8 \cdot 8=$

This expressioin equals which of the following expressions? Choose all that apply:

1 $8^3 \cdot 8^1$

2 $\frac{1}{8^{-4}}$

A. 1 only
B. 2 only
C. Neither
D. Both

9 $\frac{1}{9} \cdot \frac{1}{9} \cdot \frac{1}{9}=$

This expressioin equals which of the following expressions? Choose all that apply:

1 $\left(\frac{1}{9}\right)^3$

2 $\frac{9^2}{9^5}$

A. 1 only
B. 2 only
C. Neither
D. Both

10 $3^{-5} \cdot 3^{-7}=$

This expressioin equals which of the following expressions? Choose all that apply:

1 3^{35}

2 $\left(\frac{1}{3}\right)^{12}$

A. 1 only
B. 2 only
C. Neither
D. Both

11 $(8^3)^{-4}=$

This expressioin equals which of the following expressions? Choose all that apply:

1 8^{-12}

2 $\frac{1}{8^{-12}}$

A. 1 only
B. 2 only
C. Neither
D. Both

12 Simplify the following radical:

$\sqrt{75}$

A. There is no square root of 75
B. $5 \cdot 3$
C. $\sqrt{5} \cdot \sqrt{3}$
D. $5\sqrt{3}$

13 Simplify the following radical:

$\sqrt{\frac{1}{18}}$

A. There is no square root of $\frac{1}{18}$
B. $\frac{1}{3} \cdot \frac{1}{2}$
C. $\frac{1}{3} \cdot \sqrt{\frac{1}{2}}$
D. $\sqrt{\frac{1}{3}}\sqrt{\frac{1}{2}}$

* Unity's Basic 25 * Assessment ID: dna.15486 ib.1077078

Directions: Answer the following question(s).

14 $\frac{1}{4}+\frac{1}{3}=$

A. $\frac{2}{7}$
B. $\frac{2}{12}$
C. $\frac{7}{12}$
D. $\frac{1}{7}$

15 $3\frac{1}{4}-1\frac{1}{2}=$

A. $2\frac{1}{2}$
B. $-2\frac{1}{4}$
C. $\frac{8}{2}$
D. $\frac{7}{4}$

16 $\frac{3}{4} \cdot \left(-\frac{1}{5}\right)=$

A. $\frac{3}{20}$
B. $-\frac{3}{20}$
C. $\frac{4}{15}$
D. $-\frac{4}{15}$

17 $-\frac{3}{5} \div \frac{1}{3}=$

A. $-\frac{3}{15}$
B. $\frac{15}{3}$
C. $-\frac{15}{3}$
D. $-\frac{9}{5}$

18 Put these values in order from smallest to largest:

I) 3.22 II) 3.02 III) 3.209

A. III, II, I
B. III, I, II
C. II, I, III
D. II, III, I

19 $0.25 \div 0.05=$

A. 0.05
B. 0.5
C. 5.0
D. 50.0

20 The two legs of a right triangle are 4 inches and 5 inches long.

What is the area of the triangle measured in square inches?

A. 10
B. 16
C. 18
D. 20

* Unity's Basic 25 * Assessment ID: dna.15486 ib.1077078

Directions: Answer the following question(s).

21 The radius of a circle is 4 inches. What is the area measured in square inches?

Use $\pi = 3.14$

A. 157.7
B. 50.2
C. 15.7
D. 14

22 Solve for x:

$2x-3(x-5)=10-4(2x+4)$

A. -3
B. 3
C. $-\frac{9}{7}$
D. $\frac{9}{7}$

23 What is the slope for the line through these two points?

$(-2,-3)(2,-8)$

A. $\frac{5}{4}$
B. $-\frac{5}{4}$
C. $-\frac{11}{4}$
D. $\frac{11}{4}$

24 What is the equation of the line through these two points?

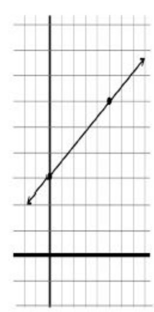
$(1,3)(-2,9)$

A. $y=2x-1$
B. $y=-2x+1$
C. $y=-2x+5$
D. $y=-2x+1$

25

What is the equation for this line:

A. $y=3x+3$
B. $y=\frac{3}{5}x+3$
C. $y=\frac{5}{3}x+3$
D. $y=-\frac{5}{3}x+3$



A. .
B. .
C. .
D. .

Appendix C

Extending Student Thinking Framework from Cengiz, Kline, Grant (2011)

Table 1 Extending student thinking framework

Extending episodes	Instructional actions
Encouraging mathematical reflection	Eliciting actions
Encouraging students to understand, compare, and generalize mathematical concepts/claims	Inviting students to share methods
	Supporting actions
Encouraging students to consider and discuss interrelationships among concepts	Suggesting an interpretation of a claim/observation
	Reminding students of goal of the discussion, the problem, or other information
Using multiple solutions to promote reflection	
Encouraging students to consider the reasonableness/validity of a claim	Repeating a claim
	Recording student thinking
Going beyond initial solution methods	Introducing different representations/contexts
Pushing individual students to try alternative solution methods for one problem situation	Extending actions
	Inviting students to:
Promoting use of more efficient solution methods for all students	Evaluate a claim or an observation
	Provide reasoning for a claim
Encouraging mathematical reasoning	Compare different methods
Encouraging students to offer a justification for their solutions/claims	Use same method for new problems
	Provide counterspeculation for a claim
Encouraging students to engage with each others' justifications	

Appendix D

Extending Student Thinking Framework from Cengiz, Kline, Grant (2011)

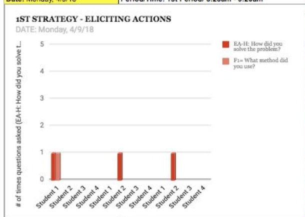
Table 3 Instructional actions that supported extending episodes

Type of instructional actions	Overall	Type of extending episode		
		Encouraging mathematical reflection	Going beyond initial solution methods	Encouraging mathematical reasoning
Eliciting actions				
Inviting students to share methods	3		3	
Extending actions				
Inviting students to				
Evaluate a claim or an observation	24	15	3	6
Provide reasoning for a claim	18	7	2	9
Compare different methods	7	6		1
Use same method for new problems	3		2	1
Provide counterspeculation for a claim	2			2
Supporting actions				
Suggesting an interpretation of a claim/observation	26	11	7	8
Reminding students of goal of the discussion, the problem, or other information	13	9	1	3
Repeating a claim	10	3	2	5
Recording student thinking	9	3	4	2
Introducing different representations/ contexts	2	2		

Appendix E

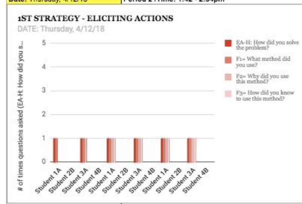
Intervention of 1st Strategy - Eliciting Actions (1 week April 9th-13th)

Date: Monday, 4/9/18 | **Period/Time:** 1st Period/ 8:28am - 9:20am



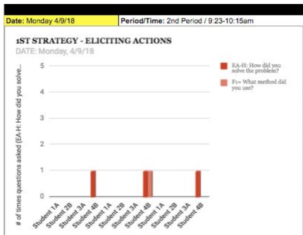
		Eliciting Actions				
		STRATEGY: Students invite each other to spafe their solution methods.				Student questions asked or comments made.
		EA-H: How did you solve the problem?	F1= What method did you use?	F2= Why did you use this method?	F3= How did you know to use this method?	
UCLA	Student 1	1	1			
	Student 2					
	Student 3					
	Student 4					
Harvard	Student 1					
	Student 2	1				
	Student 3					
	Student 4					
Cal Poly Notes: Student asked the following questions: What did you do on the other side? What did you get? Ans: I got -2 = y How did you find the second term?	Student 1					
	Student 2	1				
	Student 3					
	Student 4					

Date: Thursday, 4/12/18 | **Period 2 /Time:** 1:42 - 2:34pm



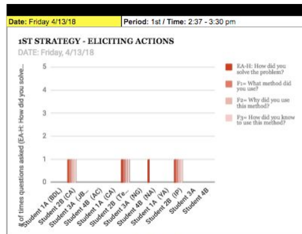
		Eliciting Actions				
		STRATEGY: Students invite each other to share their solution methods.				Student questions asked or comments made.
		EA-H: How did you solve the problem?	F1= What method did you use?	F2= Why did you use this method?	F3= How did you know to use this method?	
Stanford	Student 1A	1	1			
	Student 2B					
	Student 3A	1	1	1		
	Student 4B					
Dickinson College	Student 1A	1	1	1	1	
	Student 2B					
	Student 3A	1	1	1	1	
	Student 4B					
UC Berkely	Student 1A	1	1	1	1	
	Student 2B					
	Student 3A	1	1	1	1	
	Student 4B					

Date: Monday 4/9/18 | **Period/Time:** 2nd Period / 9:23-10:15am



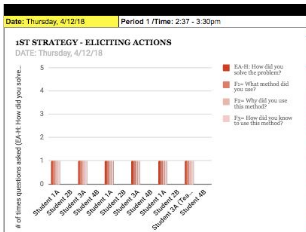
		Eliciting Actions				
		STRATEGY: Students invite each other to spafe their solution methods.				Student questions asked or comments made.
		EA-H: How did you solve the problem?	F1= What method did you use?	F2= Why did you use this method?	F3= How did you know to use this method?	
Stanford	Student 1A					
	Student 2B					
	Student 3A					
	Student 4B	1				
Dickinson College	Student 1A					
	Student 2B					
	Student 3A					
	Student 4B	1	1			
UC Berkely	Student 1A					
	Student 2B					
	Student 3A					
	Student 4B	1				

Date: Friday 4/13/18 | **Period:** 1st / **Time:** 2:37 - 3:30 pm



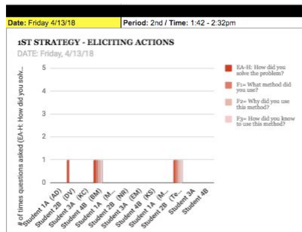
		Eliciting Actions				
		STRATEGY: Students invite each other to share their solution methods.				Student questions asked or comments made.
		EA-H: How did you solve the problem?	F1= What method did you use?	F2= Why did you use this method?	F3= How did you know to use this method?	
UCLA	Student 1A (BDL)					
	Student 2B (CA)	1	1	1	1	
	Student 3A (JBP)					
	Student 4B (AC)					
Harvard	Student 1A (CA)					
	Student 2B (Teacher)	1	1	1	1	
	Student 3A (NG)					
	Student 4B (NA)	1				
Cal Poly	Student 1A (YA)					
	Student 2B (IP)	1	1	1	1	
	Student 3A					
	Student 4B					

Date: Thursday, 4/12/18 | **Period 1 /Time:** 2:37 - 3:30pm



		Eliciting Actions				
		STRATEGY: Students invite each other to spafe their solution methods.				Student questions asked or comments made.
		EA-H: How did you solve the problem?	F1= What method did you use?	F2= Why did you use this method?	F3= How did you know to use this method?	
UCLA	Student 1A	1	1	1	1	
	Student 2B					
	Student 3A	1	1	1	1	
	Student 4B					
Harvard	Student 1A	1	1	1	1	
	Student 2B					
	Student 3A	1	1	1	1	
	Student 4B					
Cal Poly	Student 1A	1	1	1	1	
	Student 2B					
	Student 3A (Teacher) Student Absent	1	1	1	1	
	Student 4B					

Date: Friday 4/13/18 | **Period:** 2nd / **Time:** 1:42 - 2:32pm



		Eliciting Actions				
		STRATEGY: Students invite each other to share their solution methods.				Student questions asked or comments made.
		EA-H: How did you solve the problem?	F1= What method did you use?	F2= Why did you use this method?	F3= How did you know to use this method?	
Stanford	Student 1A (AD)					
	Student 2B (DV)	1				
	Student 3A (KC)					
	Student 4B (BM)	1	1	1	1	
Dickinson College	Student 1A (MH)					
	Student 2B (NR)					
	Student 3A (EM)					
	Student 4B (KS)					
UC Berkely	Student 1A (MM)					
	Student 2B (Teacher)	1	1	1	1	
	Student 3A					
	Student 4B					

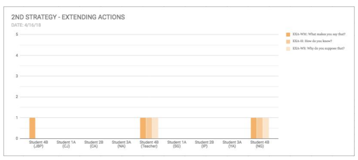
Appendix F

Intervention of 2nd Strategy-Extending Actions (2 wks April 16th-27th)

Date: Monday, 4/16/18
Period: 1st
Time: 8:28 - 9:20am

Extending Actions — STRATEGY: Students invite each other to provide reasoning for claims/solution methods.

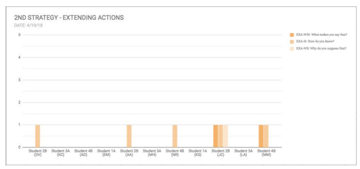
		EXA-WM: What makes you say that?	EXA-H: How do you know?	EXA-WS: Why do you suppose that?	Student questions asked or comments made.
UCLA	Student 1A (CA)				
	Student 2B (BLD)				
	Student 3A (AC)				
	Student 4B (JBP)	1			
Harvard	Student 1A (CJ)				
	Student 2B (CA)				
	Student 3A (NA)				
	Student 4B (Teacher)	1	1	1	
Cal Poly	Student 1A (SG)				
	Student 2B (IP)				
	Student 3A (YA)				
	Student 4B (NG)	1	1	1	



Date: Thursday, 4/19/18
Period: 2nd
Time: 1:42pm - 2:34pm

Extending Actions — STRATEGY: Students invite each other to provide reasoning for claims/solution methods.

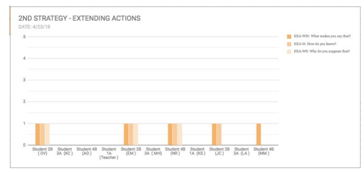
		EXA-WM: What makes you say that?	EXA-H: How do you know?	EXA-WS: Why do you suppose that?	Student questions asked or comments made.
Harvard	Student 1A (BM)				How would it be 1??
	Student 2B (DV)		1		Because we have to subtract...if you get -9 , it would be...You know what I mean?
	Student 3A (KC)				
	Student 4B (AD)				
Dickinson College	Student 1A (EM)				
	Student 2B (AA)		1		
	Student 3A (MH)				
	Student 4B (NR)		1		Ms. Stephen, is this right? Does it matter where I put the number (factors on the X-tool)?
UC Berkeley	Student 1A (KS)				(Stated claim)...because it is simple and easy to use
	Student 2B (JC)	1	1	1	
	Student 3A (LA)				(Stated claim)
	Student 4B (MM)	1	1		



Date: Monday, 4/23/18
Period/Time: 2nd Period / 9:23-10:15am

Extending Actions — STRATEGY: Students invite each other to provide reasoning for claims/solution methods.

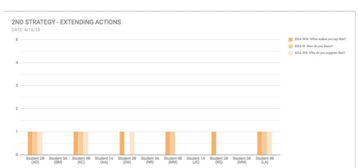
		EXA-WM: What makes you say that?	EXA-H: How do you know?	EXA-WS: Why do you suppose that?	Student questions asked or comments made.
Stanford	Student 1A (BM)				
	Student 2B (DV)	1	1	1	
	Student 3A (KC)				
	Student 4B (AD)				
Dickinson College	Student 1A (Teacher)				
	Student 2B (EM)	1	1	1	
	Student 3A (MH)				
	Student 4B (NR)	1	1	1	
UC Berkely	Student 1A (KS)				What makes me say what?
	Student 2B (JC)	1	1		
	Student 3A (LA)				
	Student 4B (MM)	1			



Date: Monday, 4/16/18
Period: 2nd
Time: 9:23-10:15am

Extending Actions — STRATEGY: Students invite each other to provide reasoning for claims/solution methods.

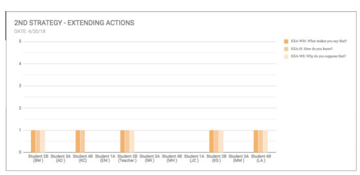
		EXA-WM: What makes you say that?	EXA-H: How do you know?	EXA-WS: Why do you suppose that?	Student questions asked or comments made.
Harvard	Student 1A (DV)				
	Student 2B (AD)	1	1	1	
	Student 3A (BM)				
	Student 4B (KC)	1	1	1	
Dickinson College	Student 1A (AA)				
	Student 2B (EM)	1		1	
	Student 3A (NR)				
	Student 4B (MM)	1	1		
UC Berkeley	Student 1A (JC)				
	Student 2B (KS)	1			
	Student 3A (MM)				
	Student 4B (LA)	1	1	1	



Date: Friday 4/20/18
Period: 2nd
Time: 1:42 - 2:34pm

Extending Actions — STRATEGY: Students invite each other to provide reasoning for claims/solution methods.

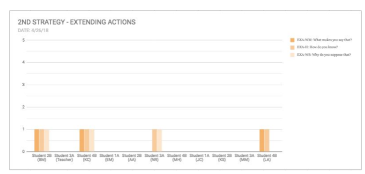
		EXA-WM: What makes you say that?	EXA-H: How do you know?	EXA-WS: Why do you suppose that?	Student questions asked or comments made.
Stanford	Student 1A (DV)				
	Student 2B (BM)	1	1	1	
	Student 3A (AD)				
	Student 4B (KC)	1	1		
Dickinson College	Student 1A (EM)				
	Student 2B (Teacher)	1	1	1	
	Student 3A (NR)				1 times 16, 2 times 8...
	Student 4B (MH)				
UC Berkeley	Student 1A (JC)				
	Student 2B (KS)	1	1	1	
	Student 3A (MM)				
	Student 4B (LA)	1	1	1	



Date: Thursday, 4/26/18
Period 2 /
Time: 1:42 - 2:34pm

Extending Actions — STRATEGY: Students invite each other to provide reasoning for claims/solution methods.

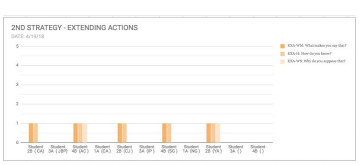
		EXA-WM: What makes you say that?	EXA-H: How do you know?	EXA-WS: Why do you suppose that?	Student questions asked or comments made.
Stanford	Student 1A (DV)				
	Student 2B (BM)	1	1	1	
	Student 3A (Teacher)				
	Student 4B (KC)	1	1	1	
Dickinson College	Student 1A (EM)				
	Student 2B (AA)				
	Student 3A (NR)		1	1	
	Student 4B (MH)				
UC Berkely	Student 1A (JC)				
	Student 2B (KS)				
	Student 3A (MM)				
	Student 4B (LA)	1	1		



Date: Thursday, 4/19/18
Period: 1st
Time: 2:37 - 3:30pm

Extending Actions — STRATEGY: Students invite each other to provide reasoning for claims/solution methods.

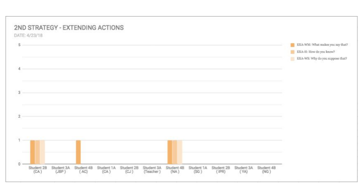
		EXA-WM: What makes you say that?	EXA-H: How do you know?	EXA-WS: Why do you suppose that?	Student questions asked or comments made.
UCLA	Student 1A (BLD)				
	Student 2B (CA)	1	1		Did you finish it? What did you get? What is the student's claim?
	Student 3A (JBP)				
	Student 4B (AC)	1	1	1	
Harvard	Student 1A (CA)				
	Student 2B (CJ)	1	1		
	Student 3A (IP)				
	Student 4B (SG)	1	1		
Cal Poly	Student 1A (NG)				
	Student 2B (YA)	1	1	1	What"s this? Did you put the answer?
	Student 3A ( )				
	Student 4B ( )				



Date: Monday, 4/23/18
Period: 1st
Time: 2:37 - 3:30pm

Extending Actions — STRATEGY: Students invite each other to provide reasoning for claims/solution methods.

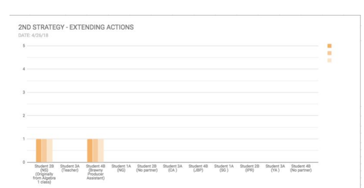
		EXA-WM: What makes you say that?	EXA-H: How do you know?	EXA-WS: Why do you suppose that?	Student questions asked or comments made.
UCLA	Student 1A (BLD)				
	Student 2B (CA)	1	1	1	
	Student 3A (JBP)				
	Student 4B (AC)	1			
Harvard	Student 1A (CA)				Question to teacher: Is this correct?
	Student 2B (CJ)				
	Student 3A (Teacher)				
	Student 4B (NA)	1	1	1	
Cal Poly	Student 1A (SG)				
	Student 2B (IPR)				
	Student 3A (YA)				
	Student 4B (NG)				



Date: Thursday, 4/26/18
Period 1
Time: 2:37 - 3:30pm
(Filmed for Brawny commercial - Teacher Appreciation)

Extending Actions — STRATEGY: Students invite each other to provide reasoning for claims/solution methods.

		EXA-WM: What makes you say that?	EXA-H: How do you know?	EXA-WS: Why do you suppose that?	Student questions asked or comments made.
Volunteer Students *from other math classes(Algebra B 2nd period and Algebra 1) for filming the Brawny commericial.*	Student 1A (KC) (Originally from 2nd period)				
	Student 2B (NS) (Originally from Algebra 1 class)	1	1	1	
	Student 3A (Teacher)				
	Student 4B (Brawny Producer Assistant)	1	1	1	
Harvard	Student 1A (NG)				Completed solving the problem
	Student 2B (No partner)				
	Student 3A (CA)				
	Student 4B (JBP)				
Cal Poly	Student 1A (SG)				
	Student 2B (IPR)				
	Student 3A (YA)				
	Student 4B (No partner)				



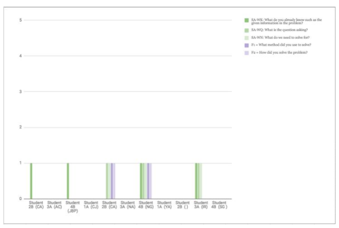
Appendix G

Intervention of 3rd Strategy - Supporting Actions (2 weeks April 30th-May 11th)

Date: Monday, 4/30/18
Period: 1st Period
Time: 8:28am - 9:20am

Supporting Actions
STRATEGY: Students remind each other of the goal of the discussion, the problem, or other information

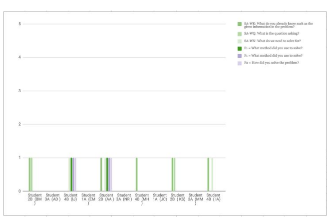
		SA-WK: What do you already know such as the given information in the problem?	SA-WQ: What is the question asking?	SA-WN: What do we need to solve for?	F1 = What method did you use to solve?	F2 = How did you solve the problem?	Student questions asked or comments made.
	Student 1A (BDL)						
	Student 2B (CA)	1					
	Student 3A (AC)						
UCLA	Student 4B (JBP)	1					
	Student 1A (CJ)						
	Student 2B (CA)		1	1	1	1	
	Student 3A (NA)						
Harvard	Student 4B (NG)	1	1	1	1	1	
	Student 1A (YA)						
	Student 2B ( )						
	Student 3A (IR)	1	1	1			Is this right? (to Teacher)
Cal Poly	Student 4B (SG)						



Date: Monday 4/30/18
Period: 2nd
Time: 9:23-10:15am

Supporting Actions
STRATEGY: Students remind each other of the goal of the discussion, the problem, or other information

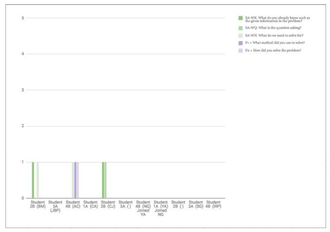
		SA-WK: What do you already know such as the given information in the problem?	SA-WQ: What is the question asking?	SA-WN: What do we need to solve for?	F1 = What method did you use to solve?	F2 = How did you solve the problem?	Student questions asked or comments made.
	Student 1A (DV)						
	Student 2B (BM)	1	1				
	Student 3A (AD)						
Stanford	Student 4B (IJ)			1	1	1	
	Student 1A (EM)						
	Student 2B (AA)	1		1	1	1	
	Student 3A (NR)						
Dickinson College	Student 4B (MH)	1					
	Student 1A (JC)						
	Student 2B (KS)	1	1				
	Student 3A (MM)						
UC Berkely	Student 4B (A)	1		1			Isn't the final question like the second question?



Date: Thursday, 5/3/18
Period: 1st
Time: 2:37 - 3:30pm

Supporting Actions
STRATEGY: Students remind each other of the goal of the discussion, the problem, or other information

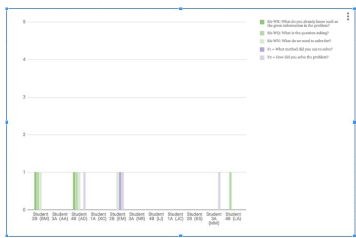
		SA-WK: What do you already know such as the given information in the problem?	SA-WQ: What is the question asking?	SA-WN: What do we need to solve for?	F1 = What method did you use to solve?	F2 = How did you solve the problem?	Student questions asked or comments made.
	Student 1A (CA)						
	Student 2B (BM)	1		1			
	Student 3A (JBP)						
UCLA	Student 4B (AC)			1	1	1	
	Student 1A (CA)						
	Student 2B (CJ)	1	1				
	Student 3A ( )						
Harvard	Student 4B (NG)						
	Student 1A (YA)						
	Student 2B ( )						
	Student 3A (SG)						
Cal Poly	Student 4B (IRP)						



Date: Thursday, 5/3/18
Period 2
Time: 1:42 - 2:34pm

Supporting Actions
STRATEGY: Students remind each other of the goal of the discussion, the problem, or other information

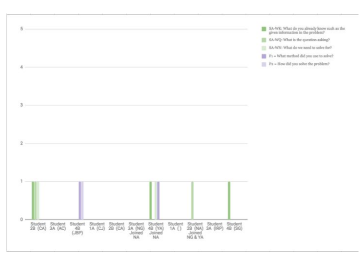
		SA-WK: What do you already know such as the given information in the problem?	SA-WQ: What is the question asking?	SA-WN: What do we need to solve for?	F1 = What method did you use to solve?	F2 = How did you solve the problem?	Student questions asked or comments made.
	Student 1A (DV)						
	Student 2B (BM)	1	1	1			
	Student 3A (AA)						
Stanford	Student 4B (AD)	1	1	1		1	
	Student 1A (KC)						
	Student 2B (EM)			1	1	1	
	Student 3A (NR)						
Dickinson College	Student 4B (IJ)						
	Student 1A (JC)						
	Student 2B (KS)						
	Student 3A (MM)					1	
UC Berkely	Student 4B (LA)		1				



Date: Friday 5/4/18
Period: 1st
Time: 2:37- 3:30 pm

Supporting Actions
STRATEGY: Students remind each other of the goal of the discussion, the problem, or other information

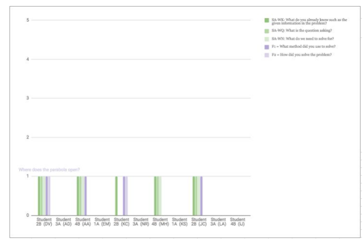
		SA-WK: What do you already know such as the given information in the problem?	SA-WQ: What is the question asking?	SA-WN: What do we need to solve for?	F1 = What method did you use to solve?	F2 = How did you solve the problem?	Student questions asked or comments made.
	Student 1A (BM)						
	Student 2B (CA)	1	1	1			
	Student 3A (AC)						
UCLA	Student 4B (JBP)				1	1	
	Student 1A (CJ)						
	Student 2B (CA)						
	Student 3A (NG)						
Harvard	Student 4B (YA)	1		1	1		
	Student 1A ( )						
	Student 2B (NA)		1				
	Student 3A (IRP)						
Cal Poly	Student 4B (SG)	1					



Date: Friday 5/4/18
Period: 2nd
Time: 1:42 - 2:34pm

Supporting Actions
STRATEGY: Students remind each other of the goal of the discussion, the problem, or other information

		SA-WK: What do you already know such as the given information in the problem?	SA-WQ: What is the question asking?	SA-WN: What do we need to solve for?	F1 = What method did you use to solve?	F2 = How did you solve the problem?	Student questions asked or comments made.
	Student 1A (BM)						
	Student 2B (DV)	1	1	1	1	1	Where does the parabola open?
	Student 3A (AD)						
Stanford	Student 4B (AA)	1	1	1	1		
	Student 1A (EM)						Where does the parabola open?
	Student 2B (KC)	1			1	1	
	Student 3A (NR)						Did you solve it using Ms. Stephen's method?
Dickinson College	Student 4B (MH)	1	1	1			
	Student 1A (KS)						
	Student 2B (JC)	1	1	1	1		
	Student 3A (LA)						
UC Berkely	Student 4B (IJ)						



Date: Monday, 5/7/18
Period: 1st
Time: 8:28am - 9:20am

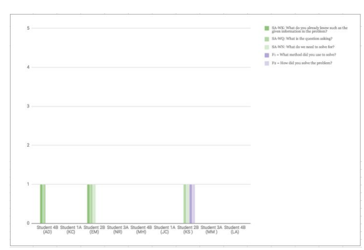
Supporting Actions
STRATEGY: Students remind each other of the goal of the discussion, the problem, or other information

		SA-WK: What do you already know such as the given information in the problem?	SA-WQ: What is the question asking?	SA-WN: What do we need to solve for?	F1 = What method did you use to solve?	F2 = How did you solve the problem?	Student questions asked or comments made.
	Student 1A (CA)						
	Student 2B (BDL)						
	Student 3A (JBp)						
UCLA	Student 4B (CA)						
	Student 1A (CJ)						
	Student 2B (AC)						
	Student 3A (NA)						
Harvard	Student 4B (NG)			1			
	Student 1A (SG)						(Asked Teacher) Is this right?
	Student 2B (IRP)						
	Student 3A (YA) Joined BDL & CA						
Cal Poly	Student 4B ( )						

Date: Monday 5/7/18
Period: 2nd
Time: 9:23-10:15am

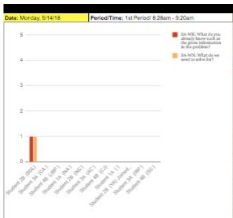
Supporting Actions
STRATEGY: Students remind each other of the goal of the discussion, the problem, or other information

		SA-WK: What do you already know such as the given information in the problem?	SA-WQ: What is the question asking?	SA-WN: What do we need to solve for?	F1 = What method did you use to solve?	F2 = How did you solve the problem?	Student questions asked or comments made.
	Student 1A (DV)						Yes
	Student 2B (BM)						So we just have to plug these in?
	Student 3A (AA)						That's the vertex, it goes in the middle.
Stanford	Student 4B (AD)	1	1				So what do I have to do
	Student 1A (KC)						
	Student 2B (EM)	1	1	1			
	Student 3A (NR)						
Dickinson College	Student 4B (MH)						
	Student 1A (JC)						
	Student 2B (KS)		1	1	1	1	
	Student 3A (MM)						
UC Berkely	Student 4B (LA)						(Asked Teacher)How do I find the x values? (T answers) Did you find the vertex?

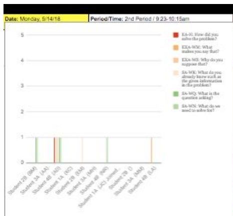


Appendix H

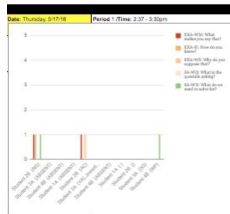
Intervention of All Strategies (1 week May 14th-18th)



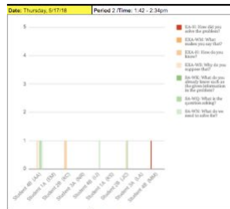
Date: Monday, 5/14/18	Period/Time: 1st Period/ 8:28am - 9:20am	Eliciting Actions	Extending Actions			Supporting Actions			
		STRATEGY: Students invite each other to spafe their solution methods.	STRATEGY: Students invite each other to provide reasoning for claims/solution methods.			STRATEGY: Students remind each other of the goal of the discussion, the problem , or other information			Student questions asked or comments made.
		EA-H: How did you solve the problem?	EXA-WM: What makes you say that?	EXA-H: How do you know?	EXA-WS: Why do you suppose that?	SA-WK: What do you already know such as the given information in the problem?	SA-WQ: What is the question asking?	SA-WN: What do we need to solve for?	
	Student 1A (CA)								How did you get zero?
	Student 2B (BDL)					1		1	
	Student 3A (CA)								
UCLA	Student 4B (JBP)								How did they get 3?
	Student 1A (NA)								
	Student 2B (NG)								
	Student 3A (AC)								
Harvard	Student 4B (CJ)								
	Student 1A ( )								
	Student 2B (YA) Joined NG & NA								
	Student 3A (IRP)								
Cal Poly	Student 4B (SG)								



Date: Monday, 5/14/18	Period/Time: 2nd Period / 9:23-10:15am	Eliciting Actions	Extending Actions			Supporting Actions			
		STRATEGY: Students invite each other to spafe their solution methods.	STRATEGY: Students invite each other to provide reasoning for claims/solution methods.			STRATEGY: Students remind each other of the goal of the discussion, the problem , or other information			Student questions asked or comments made.
		EA-H: How did you solve the problem?	EXA-WM: What makes you say that?	EXA-H: How do you know?	EXA-WS: Why do you suppose that?	SA-WK: What do you already know such as the given information in the problem?	SA-WQ: What is the question asking?	SA-WN: What do we need to solve for?	
	Student 1A (DV)								Where did you get 9 from?
	Student 2B (BM)						1	1	
	Student 3A (AA)								She squared it, that is how she(Teacher) got 9 and then she subtracted it. (Note: Student walked over to DV and showed him how he solved for the vertex.)
Harvard	Student 4B (AD)	1			1	1	1	1	
	Student 1A (KC)								How do you know how to do that? Oh you...What is this for? That's the vertex? How did you get the vertex out of -2? Where did you get the -2 from?
	Student 2B (EM)					1			It's the vertex. The k is like a y. We solved one (pair of points) now we need to find the other two(pair of points)
	Student 3A (MH)								
Dickinson College	Student 4B (NR)						1		I dont get how we got a -3 (vertex in Teacher model. Note: Teacher assits studemnt to understand)
	Student 1A (JC) Joined NR & MH								(Note: Student walked over to DV to confirm if he solved for the vertex correctly. DV confirmed that he did)
	Student 2B ()								
	Student 3A (MM)								
UC Berkely	Student 4B (LA)		1						What's your claim?



Date: Thursday, 5/17/18	Period 1 /Time: 2:37 - 3:30pm	Eliciting Actions	Extending Actions			Supporting Actions			
		STRATEGY: Students invite each other to spafe their solution methods.	STRATEGY: Students invite each other to provide reasoning for claims/solution methods.			STRATEGY: Students remind each other of the goal of the discussion, the problem , or other information			Student questions asked or comments made.
		EA-H: How did you solve the problem?	EXA-WM: What makes you say that?	EXA-H: How do you know?	EXA-WS: Why do you suppose that?	SA-WK: What do you already know such as the given information in the problem?	SA-WQ: What is the question asking?	SA-WN: What do we need to solve for?	
	Student 1A (BDL)								
	Student 2B (NG)		1	1				1	
	Student 3A (ABSENT)								
UCLA	Student 4B (ABSENT)								
	Student 1A (ABSENT)								
	Student 2B (AC)		1		1		1		
	Student 3A (YA) Joined AC								
Harvard	Student 4B (ABSENT)								
	Student 1A ( )								
	Student 2B ()								
	Student 3A (SG)								
Cal Poly	Student 4B (IRP)							1	



Date: Thursday, 5/17/18	Period 2 /Time: 1:42 - 2:34pm	Eliciting Actions	Extending Actions			Supporting Actions			
		STRATEGY: Students invite each other to spafe their solution methods.	STRATEGY: Students invite each other to provide reasoning for claims/solution methods.			STRATEGY: Students remind each other of the goal of the discussion, the problem , or other information			Student questions asked or comments made.
		EA-H: How did you solve the problem?	EXA-WM: What makes you say that?	EXA-H: How do you know?	EXA-WS: Why do you suppose that?	SA-WK: What do you already know such as the given information in the problem?	SA-WQ: What is the question asking?	SA-WN: What do we need to solve for?	
	Student 1A (BM)								
	Student 2B (DV)								
	Student 3A (AD)								
Harvard	Student 4B (AA)			1		1	1	1	
	Student 1A (EM)								The best method to use is f(x)...
	Student 2B (KC)		1	1					
	Student 3A (NR)								
Dickinson College	Student 4B (IJ)						1		
	Student 1A (KS)								
	Student 2B (JC)				1	1			
	Student 3A (LA)								
UC Berkely	Student 4B (MM)	1							

Appendix I

Use of challenging Tasks from Clarke, Roche, Cheeseman, van der Schans, 2014/2015

I did a division question correctly for homework, but the printer ran out of ink.
I remember it looked like

_ _ 4 ÷ _ = _ 4

What might be the digits that did not get printed?
(Give as many answers as you can)

Figure 3. From "Missing Number Division".

Work out which card is better value. Do this in two different ways.
Explain your thinking clearly.

Figure 2. From Music Cards (Roche & Clarke, 2013, p. 225)

Appendix J

Example of an ALL-Strategies Lesson Plan Modeled by Teacher

ALL-STRATEGIES: Supporting actions, Extending actions, and Eliciting actions. (Teacher Models)
PURPOSE: Teacher will model how to remind each other of the goal of the discussion, the problem, or other information, provide reasoning for a claim, and invite each other to share their methods when '*Writing a quadratic function given its zeros*'.
Teacher -Whole Class

- **Teacher** will solve the challenging math problem, first, then respond to the questions asked by **Student 1, 2, 3, 4, 5, 6 & 7**.
- **Student 1, 2, 3, 4, 5, 6 & 7** will ask the following questions below to the Teacher and record responses.

(Friday 5-18-18)

Applying the quadratic formula: Decimal answers

QUESTION

Use the quadratic formula to solve for x.

$$4x^2 = -8x - 1$$

Round your answer to the nearest hundredth.
If there is more than one solution, separate them with commas.

Teacher: (practice solving challenging math problem)

➢ **3RD STRATEGY: Supporting actions**

Student 1: **What do you already know, such as the given information in the problem?**

Teacher: ______

Student 2: **What is the question asking?**

Teacher: ______

Student 3: **What do we need to solve for?**

Teacher: ______

➢ **2ND STRATEGY: Extending actions**

Teacher CLAIM: ______

Student 4: What makes you say that?

Teacher: ______

Student 5: How do you know?

Teacher: ______

Student 6: Why do you suppose that?

Teacher: ______

➢ **1ST STRATEGY: Eliciting actions**

Student 7: "How did you solve the problem?"

Teacher: ______

Appendix K

Example of an ALL-Strategies Lesson Plan Student Practice w/ Teacher Feedback Rubric

ALL-STRATEGIES: **Supporting actions**, **Extending actions**, and **Eliciting actions.** (Student A & B Partners)
PURPOSE: Students will practice reminding each other of the goal of the discussion, the problem, or other information, providing reasoning for a claim, and inviting each other to share their methods when '*Applying the quadratic formula: Decimal answers*'.
Student A & B Partner Roles:

- **Student A** will solve the challenging math problem, first, then respond to the questions asked by **Student B**.
- **Student B** will solve the challenging math problem, first then ask **Student B** the following questions and record responses.

(Friday 5-18-18)

Student A & B: (practice solving challenging math problem)	**Student A & B:** (more space to practice solving challenging math problem)
Applying the quadratic formula: Decimal answers QUESTION Use the quadratic formula to solve for x. $2x^2-9x=-5$ Round your answer to the nearest hundredth. If there is more than one solution, separate them with commas.	

Teacher Feedback: I heard student(s) ask the following questions to each other...

3RD STRATEGY: Supporting actions (Student A & B Group Partners)	**2ND STRATEGY: Extending actions** (Student A & B Group Partners)	**1ST STRATEGY: Eliciting actions** (Student A & B Group Partners)
❏ **What do you already know, such as the given information in the problem?** ❏ **What is the question asking?** ❏ **What do we need to solve for?** *FOLLOW-UP QUESTIONS:* ❏ *How did you know to use this method to solve?* ❏ *How did you solve the problem?*	**Student A:** CLAIM:_______________________ ______________________________ ______________________________ ______________________________ ❏ **What makes you say that?** ❏ **How do you know?** ❏ **Why do you suppose that?**	❏ **"How did you solve the problem?"** *FOLLOW UP QUESTIONS:* ❏ *What method did you use?* ❏ *Why did you use that method?* ❏ *How did you know to use that method?*